FUNDAMENTALS OF FLUID MECHANICS

Chapter 1

Basic Properties of Fluids

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09/14/2009

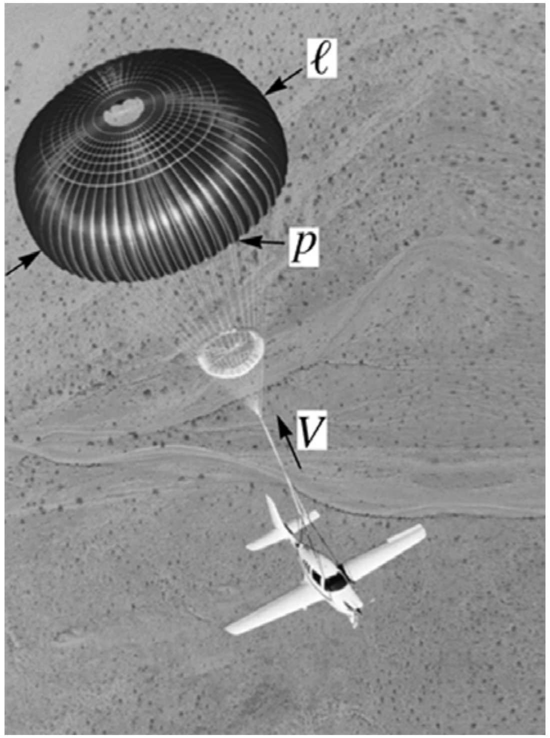
MAIN TOPICS

- Some Characteristics of Fluids
- Dimensions and Units
- Analysis of Fluid Behaviors
- Ideal Gas Law
- Fluid Properties
- Compressibility of Fluids
- Vapor Pressure
- Surface Tension

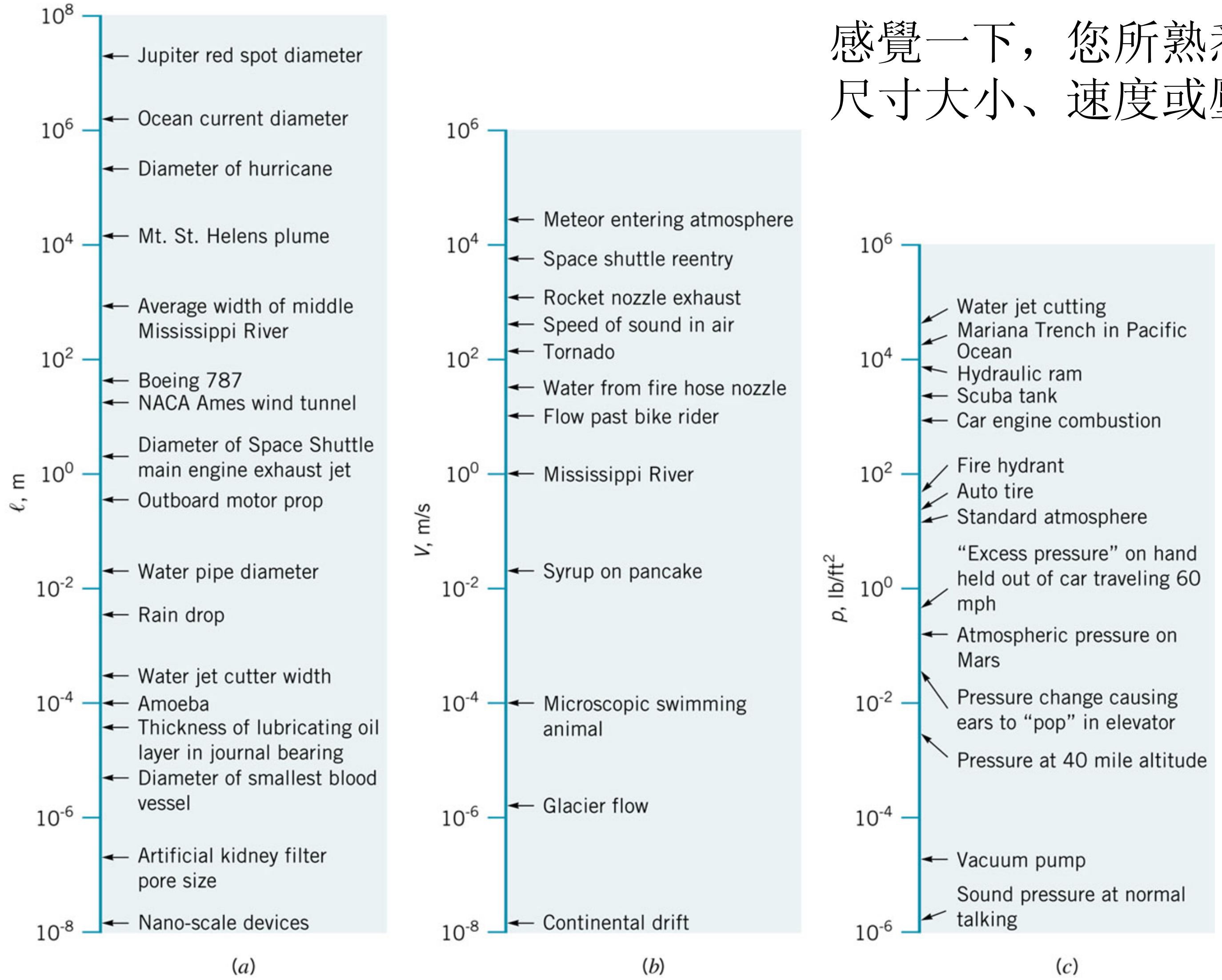
Characteristic Values of Fluids$^{1/2}$

- Size
- Speed
- Pressure

流體力學的討論，涉及的參數很多，尺寸、速度、壓力、溫度、力等等。您對這些參數的大或小？高或低？有什麼感覺？



Characteristic Values of Fluids$^{2/2}$



感覺一下，您所熟悉的一些現象，它的尺寸大小、速度或壓力的相對關係！

Mt. St. Helens火山爆發

E. coli大腸桿菌游動

Characteristics of Fluids

Characteristics of Fluids

- What's a Fluid ?
- What's difference between a solid and a fluid ?

Definition of Fluid

- Fluids comprise the liquid and gas (or vapor) phase of the physical forms.
- A fluid is a substance that deforms continuously under the application of a shear stress no matter how small the shear stress may be.
- A **shearing stress** is created whenever a tangential force acts on a surface.

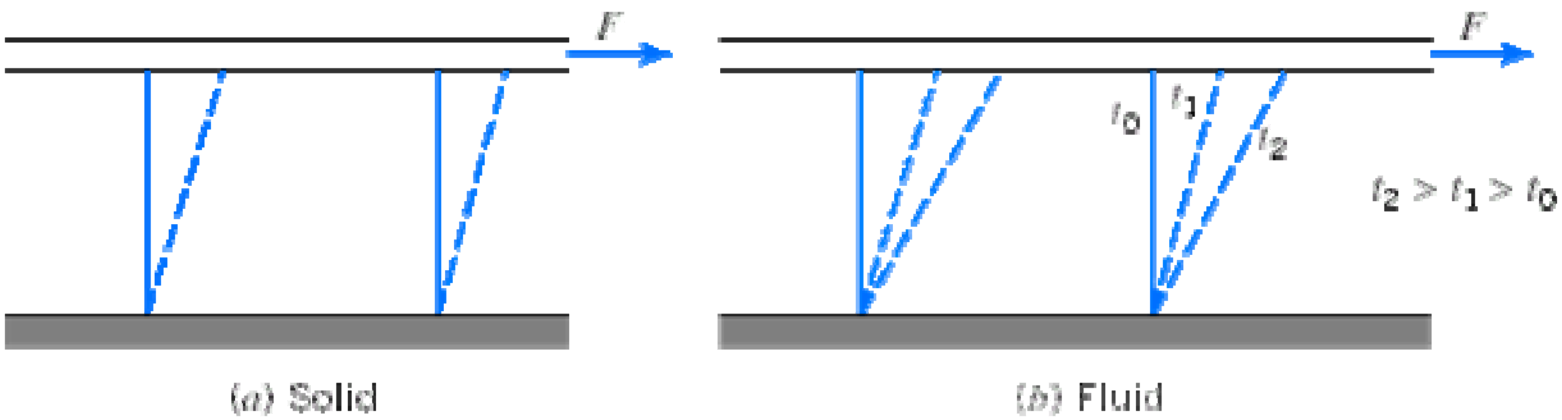
Shearing stress來自與面相切的作用力。

流體可分成液體與氣體。
流體在剪應力（不管多小）作用下可產生連續性的變形。

Fluid and Solid 1/4

施加Shear force，觀察固體與液體。

- When a constant shear force is applied:
  - ⇨Solid deforms or bends
  - ⇨Fluid continuously deforms.



Fluid and Solid 2/4

- Vague idea　從模糊的觀點，看固體與流體的不同。
  - ⇨Fluid is soft and easily deformed.
  - ⇨Solid is hard and not easily deformed.
- Molecular structure
  - ⇨Solid has densely spaced molecules with large intermolecular cohesive force allowed to maintain its shape. 從分子結構的觀點，看待固體與流體。
    固體的分子間距較小，分子間的結合力較大，足以維持其形狀。

Fluid and Solid 3/4

⇨ Liquid has further apart spaced molecules, the intermolecular forces are smaller than for solids, and the molecules have more freedom of movement. At normal temperature and pressure, **the spacing is on the order of 10^{-7} mm**. The **number of molecules** per cubic millimeter is on the order of **10^{21}** .

液體的分子間距稍大，分子間的結合力較固體低，其自由活動能力較強。在正常溫度與壓力下，分子間距約10^{-7} mm。每立方厘米的分子數量約爲10^{21}。

Fluid and Solid 4/4

- Gases have even greater molecular spacing and freedom of motion with negligible cohesive intermolecular forces and as a consequence are easily deformed. At normal temperature and pressure, the spacing is on the order of 10^{-6} mm. The number of molecules per cubic millimeter is on the order of 10^{18} .

氣體的分子間距最大，分子間的結合力幾乎可忽略，其自由活動能力最強，最容易變形。在正常溫度與壓力下，分子間距約10^{-6} mm。每立方厘米的分子數量約為10^{18}。

Fluid? Solid ?

- Some materials, such as slurries, tar, putty, toothpaste, and so on, are not easily classified since they will behave as solid if the applied shearing stress is small, but if the stress exceeds some critical value, the substance will flow. The study of such materials is called **rheology**.

有些物質，如泥漿、焦油、油灰、牙膏等，像固體（Shearing stress小），又似流體（Shearing stress大）。
探討這類物質的學門，稱為「流變學」。

Characteristic Description

- Qualitative aspect
- Quantitative aspect

定性與定量觀點來
描述流體的性質。

Qualitative Aspect

從定性的觀點來界定性質的本質與種類，如長度、時間、應力與速度。

- Qualitative aspect serves to identify the nature, or type, of the characteristics (such as length, time, stress, and velocity) .
- Qualitative description is given in terms of certain **primary quantities, such as Length,L, time, T, mass, M, and temperature, θ**. The primary quantities are also referred to as **basic dimensions**.
- These primary quantities can then used to provide a qualitative description of any other **secondary quantity:** for example, area$\doteq L^2$,velocity $\doteq Lt^{-1}$,density $\doteq ML^{-3}$.

從定性的觀點來描述或界定性質時，用到Primary quantities 或 Secondary quantity。統稱 Physical Quantities。

Primary vs. Secondary quantities

- Primary quantities：又稱Basic dimension，包括長度L、時間T、質量M與溫度θ。
- Secondary quantity：由 Primary quantities 組合而成。如體積≒ 長度×長度×長度，速度≒長度÷時間，密度≒質量÷體積。

Quantitative Aspect

特性的數量化，數量化需要數字與標準。

- Provide a numerical measure of the characteristics.
- Require both a number and a standard.
- A standard for length might be a meter or foot, for time an hour or second, and for mass a slug or kilogram.
- Such **standards are called units**.

從定量的觀點來界定或描述性質時，會從以數值衡量該性質的角度切入。因此需要Number與Standard.　Standard? 如衡量『長度』時，是用meter或foot? 衡量『時間』時，是用hour或second?

Standards == Units，標準即為「單位」！

Primary and Secondary Quantities

- Primary quantities also referred as basic dimensions
  - Such as Length, L, time, T, mass, M, and temperature, θ .
  - Used to provide a qualitative description of any other secondary quantity.
- Secondary quantities
  - For example, area $\doteq L^2$, velocity $\doteq Lt^{-1}$, density $\doteq ML^{-3}$.
  - The symbol $\doteq$ is used to indicate the dimensions of the secondary quantity in terms of the primary quantities.

Dimension, Dimensional Homogeneity, and Units

Length、Time、Mass、Velocity、Area、Density是用來『定性』描述「性質」的 Physical Quantities ！

『定量』描述? Number + Standard。

Standard ? ＝ Units

Dimension? L, M, T, θ

System of Dimensions

- Mass[M], Length[L], time[T], and Temperature[θ] …**MLT system**
- Force[F], Length[L], time[T], and Temperature[θ] …**FLT system**
- Force[F], Mass[M], Length[L], time[T], and Temperature[θ] …**FMLT system**

因次系統的分類！

Dimensions Associated with Common Physical Quantities

常用的physical quantities與Dimension的關係。

■ TABLE 1.1

Dimensions Associated with Common Physical Quantities

	FLT System	*MLT* System
Acceleration	LT^{-2}	LT^{-2}
Angle	$F^0L^0T^0$	$M^0L^0T^0$
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Energy	FL	ML^2T^{-2}
Force	F	MLT^{-2}
Frequency	T^{-1}	T^{-1}
Heat	FL	ML^2T^{-2}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of a force	FL	ML^2T^{-2}
Moment of inertia (area)	L^4	L^4
Moment of inertia (mass)	FLT^2	ML^2
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	ML^2T^{-3}
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	$F^0L^0T^0$	$M^0L^0T^0$
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}
Temperature	Θ	Θ
Time	T	T
Torque	FL	ML^2T^{-2}
Velocity	LT^{-1}	LT^{-1}
Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Viscosity (kinematic)	L^2T^{-1}	L^2T^{-1}
Volume	L^3	L^3
Work	FL	ML^2T^{-2}

Dimensionally Homogeneous

- All theoretically derived **equations** are ***dimensionally homogeneous***– that is, the dimensions of the left side of the equation must be the same as those on the right side, and all additive separate terms have the same dimensions.
  - ⇨General homogeneous equation: valid in any system of dimensions.
  - ⇨Restricted homogeneous equation : restricted to a particular **system of dimenions**.

方程式必須符合dimensionally homogeneous的規則，不管是等號左右兩側或方程式內的相加項，均具有相同的dimensions。

Dimensionally Homogeneous

$$V = V_0 + at \Rightarrow LT^{-1} \doteq LT^{-1} + LT^{-1}$$

$$d = \frac{gt^2}{2} \xrightarrow{g\,=\,32.2\ \mathrm{ft/sec^2}} d = 16.1t^2$$

Valid only for the system of units using feet and seconds

$$d = \frac{gt^2}{2} \xrightarrow{g\,=\,9.81\ \mathrm{m/sec^2}} d = 4.90t^2$$

Valid only for the system of units using feet and seconds

NOTE：如此簡化限制於特定 system of dimensions。使用不同的system of dimensions就不見得

Systems of Units 1/2

- In addition to the qualitative description of the various quantities of interest, it is generally necessary to have a **<u>quantitative measure of any given quantity</u>**. 除定性描述外，需要有「定量」的衡量。
- For example, if we measure the width of this page in the book and say that it is 10 units wide, the statement has no meaning until the unit of length is defined. 紙的寬度是10個單位？沒有意義！長度的「單位」？
  - ► If we indicate that the units of length is a meter, and define the meter as some standard length, a unit system for length has been established. 指定長度的「單位」是 meter 時，長度的單位系統就浮現了！
- **A unit must be established for each of the remaining basic quantities**.

每一個basic quantity都必須有『Unit』！
連帶地，second quantities也有『Unit』！

Systems of Units 2/2

- British Gravitational System: B.G.
- International System: S.I.
- English Engineering: E.E.

BG system and SI system are widely used in engineering

B.G.與SI用得最廣泛！

來源

In 1960 the 11th General Conference on Weights and Measures, the international organization responsible for maintaining precise uniform standards of measurement, formally adopted the International System of Units as the international standard. This system, commonly termed SI, has been widely adopted worldwide and is widely used in the United States.

SI於1960年被國際採用，是最普及的系統。

British Gravitational System

- Length: ft
- Time: second
- Force: lb
- Temperature: ^{o}F or ^{o}R : $^{o}R = {}^{o}F + 459.67$
- Mass: slug : $1 \text{ lb} = 1 \text{ slug} \times 1 \text{ ft} / \text{sec}^2$
- Gravity: $g = 32.174 \text{ ft} / \text{sec}^2$
- Weight: $W \text{ (lb)} = m \text{ (slug)} \times g \text{ (ft} / \text{sec}^2)$

International System (SI)

- Length: m
- Time: second
- Mass: Kg
- Temperature : ^oK : $^oK = {^oC} + 273.15$
- Force: Newton: $1\ N = 1\ Kg \times 1\ m / sec^2$
- Work: Joule (J) ; $J = 1\ N \times m$
- Power: Watt (W) ; $W = J / sec = N \times m/sec$
- Gravity: $g = 9.807\ m / sec^2$
- Weight: $W\ (N) = m\ (Kg) \times g\ (m/\ sec^2)$: 1 kg-mass weights 9.81N

English Engineering (EE) System

- Mass: lbm
- Force: lbf
- Length: ft
- Time: second
- Temperature: °R (absolute temperature)

$F = ma / g_c$ ； g_c : the constant of proportionality

$1\ lbf = (lb_m \times 32.174\ ft / sec^2) / g_c$ 最複雜！

$g_c = (lb_m \times 32.174\ ft / sec^2) / lbf$

In E.E., the relationship between weight and mass :

$W = mg / gc$ Therefore, 1 slug = 32.174 lbm (when $g = g_c$)

Conversion Factor

TABLE 1.3

Conversion Factors from BG and EE Units to SI Units*

	To Convert from	to	Multiply by
Acceleration	ft/s^2	m/s^2	3.048 E − 1
Area	ft^2	m^2	9.290 E − 2
Density	lbm/ft^3	kg/m^3	1.602 E + 1
	$slugs/ft^3$	kg/m^3	5.154 E + 2
Energy	Btu	J	1.055 E + 3
	ft · lb	J	1.356
Force	lb	N	4.448
Length	ft	m	3.048 E − 1
	in.	m	2.540 E − 2
	mile	m	1.609 E + 3
Mass	lbm	kg	4.536 E − 1
	slug	kg	1.459 E + 1
Power	ft · lb/s	W	1.356
	hp	W	7.457 E + 2
Pressure	in. Hg (60 °F)	N/m^2	3.377 E + 3
	lb/ft^2 (psf)	N/m^2	4.788 E + 1
	$lb/in.^2$ (psi)	N/m^2	6.895 E + 3
Specific weight	lb/ft^3	N/m^3	1.571 E + 2
Temperature	°F	°C	$T_C = (5/9)(T_F - 32°)$
	°R	K	5.556 E − 1
Velocity	ft/s	m/s	3.048 E − 1
	mi/hr (mph)	m/s	4.470 E − 1
Viscosity (dynamic)	$lb \cdot s/ft^2$	$N \cdot s/m^2$	4.788 E + 1
Viscosity (kinematic)	ft^2/s	m^2/s	9.290 E − 2
Volume flowrate	ft^3/s	m^3/s	2.832 E − 2
	gal/min (gpm)	m^3/s	6.309 E − 5

*If more than four-place accuracy is desired, refer to Appendix E.

TABLE 1.4

Conversion Factors from SI Units to BG and EE Units*

	To Convert from	to	Multiply by
Acceleration	m/s^2	ft/s^2	3.281
Area	m^2	ft^2	1.076 E + 1
Density	kg/m^3	lbm/ft^3	6.243 E − 2
	kg/m^3	$slugs/ft^3$	1.940 E − 3
Energy	J	Btu	9.478 E − 4
	J	ft · lb	7.376 E − 1
Force	N	lb	2.248 E − 1
Length	m	ft	3.281
	m	in.	3.937 E + 1
	m	mile	6.214 E − 4
Mass	kg	lbm	2.205
	kg	slug	6.852 E − 2
Power	W	ft · lb/s	7.376 E − 1
	W	hp	1.341 E − 3
Pressure	N/m^2	in. Hg (60 °F)	2.961 E − 4
	N/m^2	lb/ft^2 (psf)	2.089 E − 2
	N/m^2	$lb/in.^2$ (psi)	1.450 E − 4
Specific weight	N/m^3	lb/ft^3	6.366 E − 3
Temperature	°C	°F	$T_F = 1.8\, T_C + 32°$
	K	°R	1.800
Velocity	m/s	ft/s	3.281
	m/s	mi/hr (mph)	2.237
Viscosity (dynamic)	$N \cdot s/m^2$	$lb \cdot s/ft^2$	2.089 E − 2
Viscosity (kinematic)	m^2/s	ft^2/s	1.076 E + 1
Volume flowrate	m^3/s	ft^3/s	3.531 E + 1
	m^3/s	gal/min (gpm)	1.585 E + 4

*If more than four-place accuracy is desired, refer to Appendix E.

Dimension vs. Systems of Units

- MLT Dimension system
  - International System (kg, m, s, ^oK)
- FLT Dimension system
  - British Gravitational (lbf, ft, s, ^oR)
- FMLT Dimension system
  - English Engineering (lbf, lbm, ft, s, ^oR)

Preferred Systems of Units

- SI (kg, m, s, °K)

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m} / \text{s}^2$$

- British Gravitational (lb, ft, s, °R)

$$1 \text{ lb} = 1 \text{ slug} \cdot \text{ft} / \text{s}^2$$

$$1 \text{ slug} = 1 \text{ lb} \cdot \text{s}^2 / \text{ft}$$

Example 1.2 BG and SI Units

Example 1.2 BG and SI units

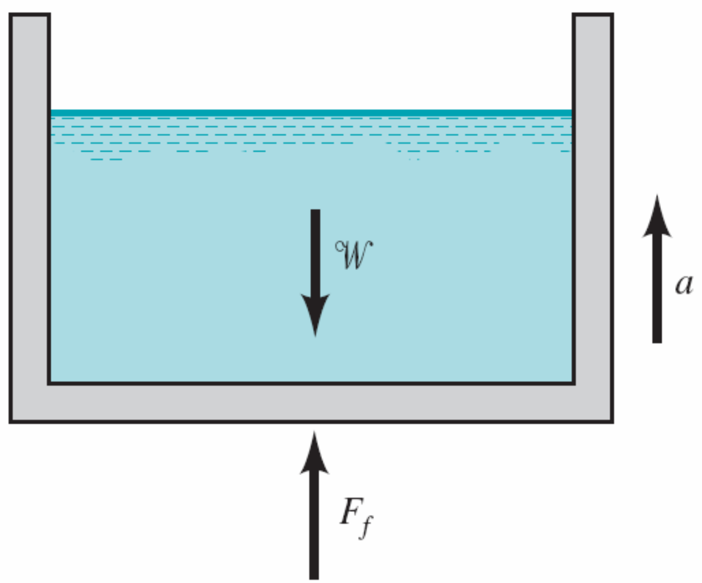
- A tank of water having a total mass of 36 kg rests on the floor of an elevator. Determine the **forces (in newtons)** that the tank exerts on the floor when the elevator is accelerating upward at 7ft/s^2.

$$\sum F = m\vec{a} \text{ or } \underbrace{F_f - W}_{合力} = ma$$

$$F_f = m(g + a)$$

$$= 36\text{kg}[9.81\text{m}/\text{s}^2 + (7\text{ft}/\text{s}^2)(0.3048\text{m}/\text{ft})]$$

$$= 430\text{kg}\cdot\text{m}/\text{s}^2 = 430\text{N}$$



$$1\ \text{N} = 1\ \text{kg}\cdot\text{m}/\text{s}^2$$

放在升降梯上，升降梯的加速度為7 ft/s^2。求Tank施加在升降梯地板的力？

Analysis of Fluid Behaviors 1/2

把描述流體運動的基本原理寫出來。

流體力學常用的基本原理

- Analysis of any problem in fluid mechanics necessarily includes statement of the basic laws governing the fluid motion. The basic laws, which applicable to any fluid, are:
  - ⇨Conservation of mass
  - ⇨Newton's second law of motion
  - ⇨The principle of angular momentum
  - ⇨The first law of thermodynamics
  - ⇨The second law of thermodynamics

Analysis of Fluid Behaviors 2/2

- NOT all basic laws are required to solve any one problem. On the other hand, in many problems it is necessary to bring into the analysis additional relations that describe the behavior of physical properties of fluids under given conditions.
- Many apparently simple problems in fluid mechanics that **cannot be solved analytically**. In such cases we must resort to more ***complicated numerical solutions and/or results of experimental tests.***

並非每一個流體力學所探討的問題都會用到所有基本原理。要帶入哪一個原理？當然要看已知條件而定。同時，也要提醒大家，多數的流力問題都沒有解析解。而是得訴諸數值解或實驗。

Measurement of Fluid Mass and Weight

- Density
- Specific weight
- Specific Gravity

衡量流體質量與重量?

Density

密度的定義：單位流體體積的質量。
密度是衡量流體『mass』的一個指標。

- The density of a fluid, designated by the Greek symbol ρ (rho), is **defined as its mass per unit volume.**
- Density is used to characterize the mass of a fluid system.
- **In the BG system ρ has units of slug/ft^3 and in SI the units are kg/m^3.** 密度的單位。
- The value of density can vary widely between different fluids, but for liquids, variations in pressure and temperature generally have only a small effect on the value of density. 密度與壓力、溫度的關係。

⇨ **The specific volume, υ , is the volume per unit mass – that is,**

$$\upsilon = 1/\rho$$

液體的密度隨溫度與壓力變化不大。

密度與比體積的關係。

Specific Weight

❖ The specific weight of a fluid, designated by the Greek symbol γ (gamma), is defined as its weight per unit volume.

$$\gamma = \rho g$$

比重量的定義：單位體積的流體的重量。

❖ Under conditions of standard gravity ($g = 9.807 m/s^2 = 32.174 ft/s^2$), water at 60ºF has a specific weight of <u>62.4lb/ft^3</u> and <u>9.80</u>kN/m^3. **The density of water is 1.94slug/ft^3 or 999kg/m^3.**

比重量的單位。

Specific Gravity

❖ The specific gravity of a fluid, designated as SG, is defined as the ratio of the density of the fluid to the density of water at some specified temperature.

$$SG = \frac{\rho}{\rho_{H_2O@4°C}}$$

比重的定義：流體密度相對於水在特定溫度下的密度的比值。

$\rho_{H_2O\,,\,4°C} = 1.94 slug/ft^3$ **or** $999 kg/m^3$. 水在4°C下的密度。

Ideal Gas Law

理想氣體定理 vs. 理想氣體

氣體的可壓縮性高於液體。壓力與溫度、密度的關係?

- ❖ Gases are highly compressible in comparison to fluids, with changes in gas density directly related to changes in pressure and temperature through the equation $p=\rho RT$. P 是絕對壓力。
- ❖ The ideal gas equation of state $p=\rho RT$ is a model that relates density to pressure and temperature for many gases under normal conditions. 理想氣體狀態方程式…描述壓力、溫度、密度的關係。
- ❖ **The pressure in the ideal gas law must be expressed as an absolute pressure which is measured relative to absolute zero pressure.**
- ❖ **The standard sea-level atmospheric pressure is 14.6996 psi (abs) or 101.33kPa (abs).** 海平面上的絕對壓力。

符合理想氣體定理者稱理想氣體，理想氣體可以理想氣體定理來描述。

Example 1.3 Ideal Gas Law

- A compressed air tank has a volume of 0.84 ft^3. When the tank is filled with air at a **gage pressure of 50 psi**, determine the density of the air and the weight of air in the tank. Assume the temperature is 70 °F and the atmospheric pressure is 14.7 psi (abs).

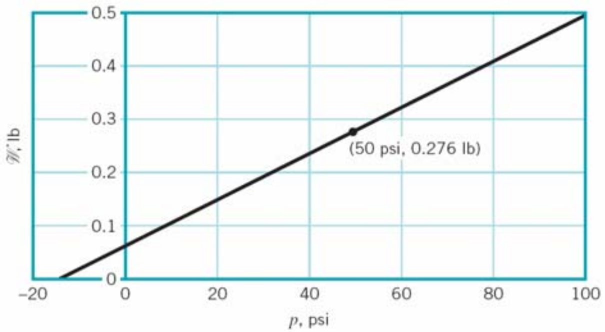
$$\rho = \frac{P}{RT} = ... = 0.0102 \ \text{slugs/ft}^3$$

$$W = \rho g(\text{volume})$$

$$= (0.0102 \ \text{slugs/ft}^3)(32.2\text{ft/s}^2)(0.84\text{ft}^3)$$

$$= 0.276\text{slug} \cdot \text{ft/s}^2 = 0.276\text{lb}$$

Pressure vs. Weight



已知壓力，透過理想氣體方程式求密度，再求出tank內氣體的重量。

Example 1.3 Ideal Gas Law NEW

- A compressed air tank has a volume of 0.024 m^3. The temperature is 20 °C and the atmospheric pressure is 101.3 kPa (abs). When the tank is filled with air at a **gage pressure of 345 kPa**, determine the density of the air and the weight of air in the tank. .

已知壓力與溫度，透過理想氣體方程式求密度，再求出tank內氣體的重量。

Viscosity

密度與比重量是衡量流體HEAVINESS的指標。

- The properties of density and specific weight are measures of the “heaviness” of a fluid.
- It is clear, however, that these properties are not sufficient to uniquely characterize how fluids behave since two fluids can have approximately the same value of density but behave quite differently when flowing.
- There is apparently some additional property that is needed to describe the “**FLUIDITY**” of the fluid.

流體的密度相近，但流動起來的差異性，要如何來描述？

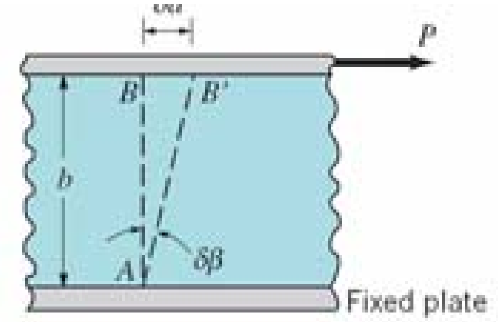
黏度是衡量流體流動性的指標。 Viscous fluids

容易流動或不容易流動？

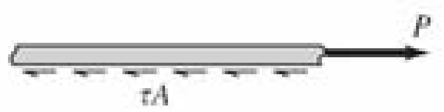
Fluidity of Fluid 1/3

如何描述流體的流動性？

- ❖ **How to describe the "fluidity" of the fluid?**
  - ➪ **The bottom plate is rigid fixed, but the upper plate is free to move.**
  - ➪ **If a solid, such as steel, were placed between the two plates and loaded with the force *P*, the top plate would be displaced through some small distance, δa.**
  - ➪ **The vertical line AB would be rotated through the small angle, δβ, to the new position AB'.**



(a)



(b)

$P = \tau A$

當兩平板間材料是固體時…

Fluidity of Fluid 2/3

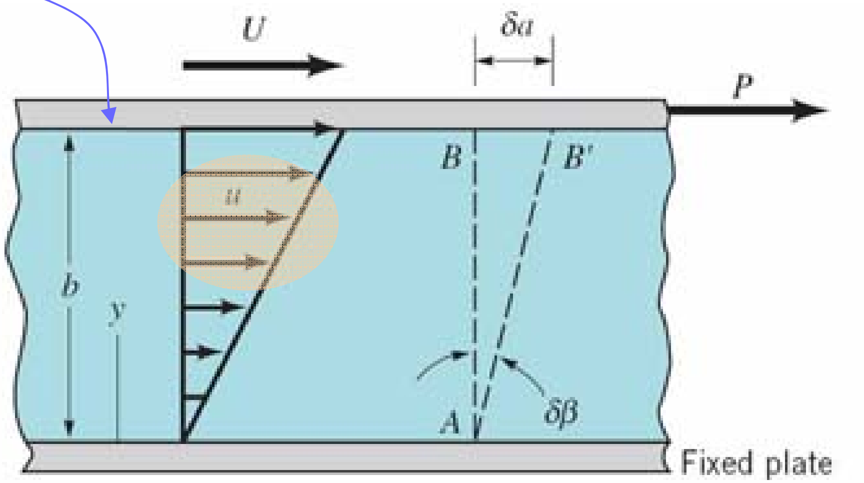
當兩平板間材料換成流體時…

❖ What happens if **the solid is replaced with a fluid** such as water?

⇨ When the force P is applied to the upper plate, it will move continuously with a velocity U.

⇨ The fluid "sticks" to the solid boundaries and is referred to as the NON-SLIP conditions.

緊貼上方平板的流體持續移動。



緊貼下方平板的流體沒有移動。

No-slip conditions

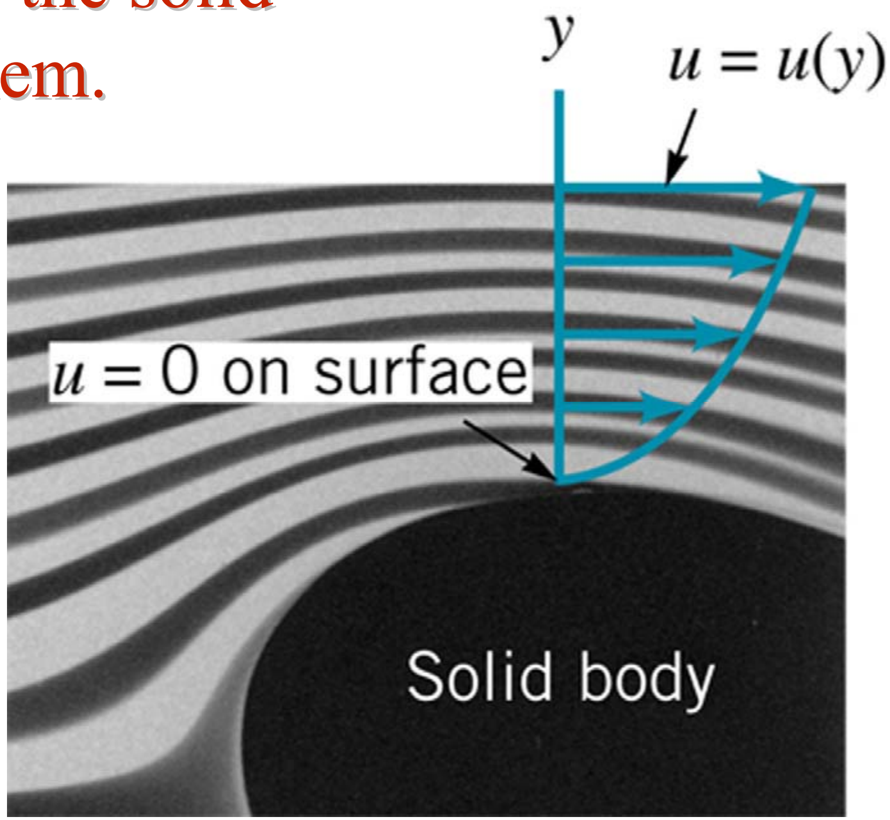
兩板間的流體如何變化？

⇨ The fluid between the two plates moves with velocity u=u(y) that would be assumed to vary linearly, u=Uy/b. In such case, the velocity gradient is **$du / dy = U / b$.**

速度梯度

Real Fluids

Real fluids, even though they may be moving, always “stick” to the solid boundaries that contain them.



Fluidity of Fluid 3/3

兩板間的流體出現什麼狀況?

- In a small time increment, δt, an imaginary vertical line AB would rotate through an angle, $\delta\beta$, so that

  $\tan\delta\beta \fallingdotseq \delta\beta = \delta a / b$

  找出δβ與b、U、δt的關係!

  Since $\delta a = U\delta t$ it follows that $\delta\beta = U\delta t / b$

  $\delta\beta ? \rightarrow \delta\beta = \delta\beta\ (P, t)$

  定義剪應變率。

- Defining the rate of shearing strain, γ, as $\dot{\gamma} = \lim_{\delta t \to 0} \frac{\delta\beta}{\delta t} = \frac{U}{b} = \frac{du}{dy}$
- The shearing stress is increased by P, the rate of shearing strain is increased in direct proportion, $\tau \propto \dot{\gamma}$ or $\tau \propto du/dy$

The **common fluids such as water, oil, gasoline, and air**. The shearing stress and rate of shearing strain can be related with a relationship

$$\tau = \mu \frac{du}{dy}$$

存在線性關係者，為牛頓流體。

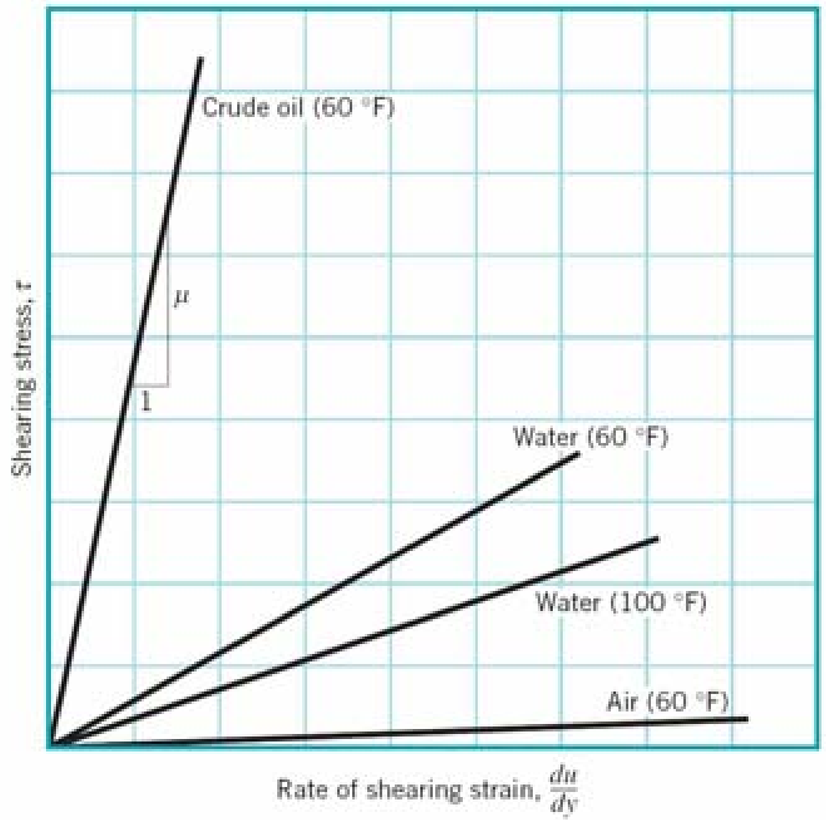
Viscosity Definition

Dynamic viscosity is the fluid property that relates shearing stress and fluid motion.

比例常數就是黏度

$$\tau = \mu \frac{du}{dy}$$

稱爲絕對黏度、動力黏度，簡稱黏度



- The constant of proportionality is designated by the **Greek symbol μ (mu) and is called the absolute viscocity, dynamic viscosity, or simply the viscosity of the fluid.**
- The viscosity depends on the particular fluid, and for a particular fluid the viscosity is also **dependent on temperature.**

黏度因流體不同而異，同一流體，其黏度受溫度變化影響。

Dimension and Unit of μ

黏度的因次與單位

- The dimension of μ : Ft/L^2 or M/Lt.
- The unit of μ :
  - In B.G. : $lbf\text{-}s/ft^2$ or slug/(ft-s)
  - In S.I. : kg/(m-s) or $N\text{-}s/m^2$ or Pa -s
  - In the Absolute Metric: poise=1g/(cm-s)

Example 1.4 Viscosity and Dimensionless Quantities

- A dimensionless combination of variables that is important in the study of viscous flow through pipes is called the *Reynolds number*, Re, defined as $\rho VD/\mu$ where ρ is the fluid density, V the mean velocity, D the pipe diameter, and μ the fluid viscosity. A newtonian fluid having a viscosity of 0.38 N·s/m^2 and a specific gravity of 0.91 flows through a 25-mm-diameter pipe with a velocity of 2.6 m/s. Determine the value of the Reynolds number using (a) SI units, and (b) BG units.

流體在管內流動。已知流體的比重0.91、黏度、速度、管徑，依定義求出Reynolds number。

$$\rho = SG\rho_{H_2O@4°C} = 0.91(1000 kg/m^3)$$

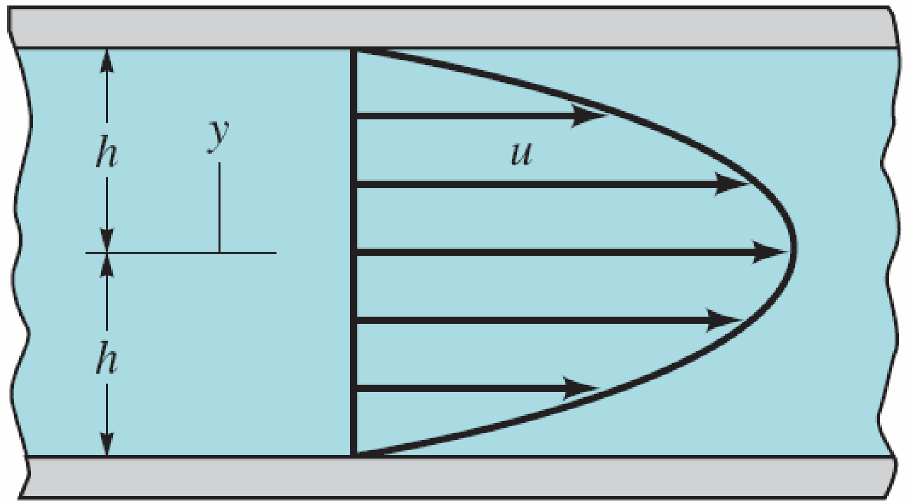
$$Re = \frac{\rho VD}{\mu} = \ldots = 156\left(kg \cdot m/s^2\right)/N = 156$$

Example 1.5 Newtonian Fluid Shear Stress

- The velocity distribution for the flow of a Newtonian fluid between two sides, parallel plates is given by the equation

平板間流動流體的速度分佈

$$u = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$$



Example 1.5 Solution

where V is the mean velocity. The fluid has a viscosity of 0.04 lb·s/ft². When V=2 ft/s and h=0.2 in. determine: (a) the shearing stress acting on the bottom wall, and (b) the shearing stress acting on a plane parallel to the walls and passing through the centerline (midplane).

$$\tau = \mu \frac{du}{dy} = -\mu \frac{3Vy}{h^2}$$

速度梯度與剪應力的關係式

$$\tau_{\text{bottom wall}} = \mu \frac{du}{dy} = -\mu \left.\frac{3Vy}{h^2}\right|_{y=-h} = 14.4 \text{lb}/\text{ft}^2$$

$$\tau_{\text{midplane}} = \mu \frac{du}{dy} = -\mu \left.\frac{3Vy}{h^2}\right|_{y=0} = 0$$

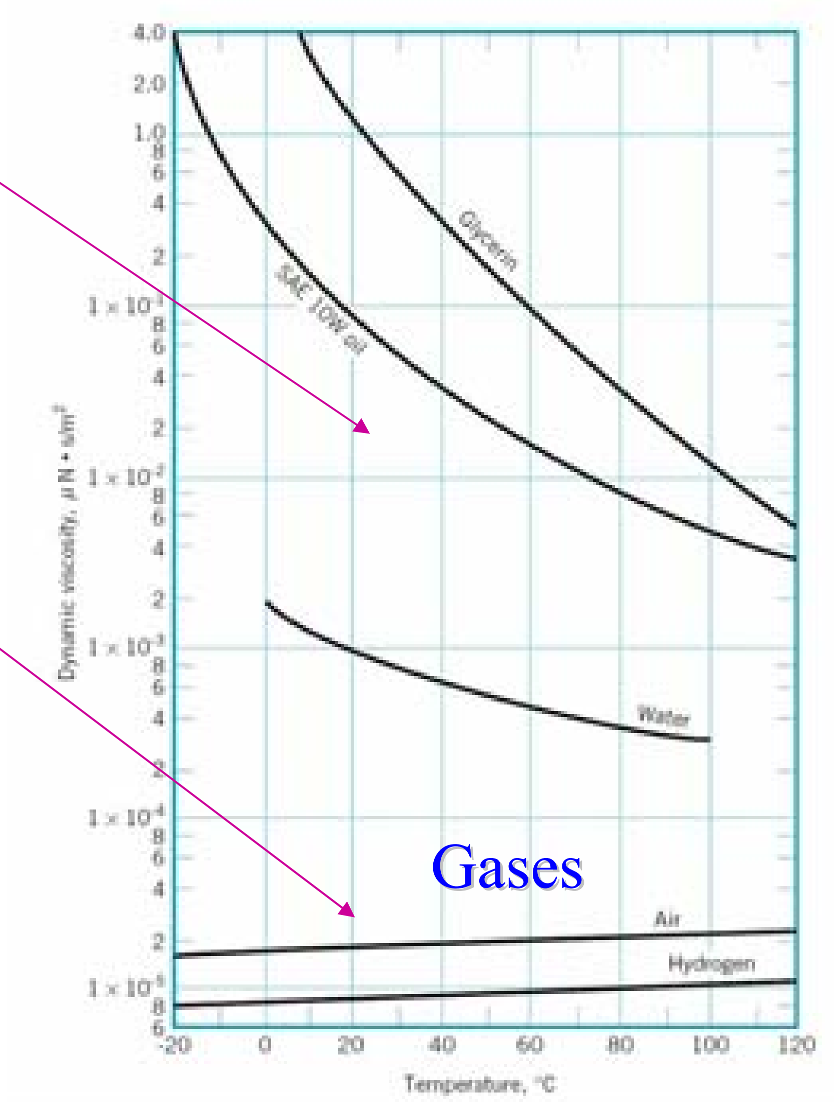
Viscosity vs. Temperature 1/3

- For fluids, the viscosity decreases with an increase in temperature.
- For gases, an increase in temperature causes an increase in viscosity.

⇨ WHY? molecular structure.

液體的溫度↑黏度↓
氣體的溫度↑黏度↑

從分子結構的觀點來解釋☛



Viscosity vs. Temperature 2/3

❖ The liquid molecules are closely spaced, with strong cohesive forces between molecules, and the resistance to relative motion between adjacent layers is related to these intermolecular force.

- ▶As the temperature increases, these cohesive force are reduced with a corresponding reduction in resistance to motion. Since viscosity is an index of this resistance, it follows that ***viscosity is reduced by an increase in temperature.***
- ▶**The Andrade's equation** $\mu = De^{B/T}$ 黏度與溫度關係

液體分子間距較緊密，分子間結合力較強，阻止層與層間相對運動的阻力與分子間結合力有關。溫度上升，分子間結合力降低，阻力自然降低。

Viscosity vs. Temperature 3/3

- **In gases,** the molecules are widely spaced and intermolecular force negligible.
  - The resistance to relative motion mainly arises due to the exchange of momentum of gas molecules between adjacent layers.
  - As the temperature increases, ***the random molecular activity increases with a corresponding increase in viscosity.***

黏度與溫度關係

  - **The Sutherland equation** $\mu = CT^{3/2} / (T+S)$

氣體分子間距較大，分子間結合力幾乎可忽略，阻止層與層間相對運動的阻力與分子間動量交換有關。溫度上升，分子間動量交換提高，阻力跟著提高。

Newtonian and Non-Newtonian Fluid

常見流體，如水、空氣、汽油等為牛頓流體。

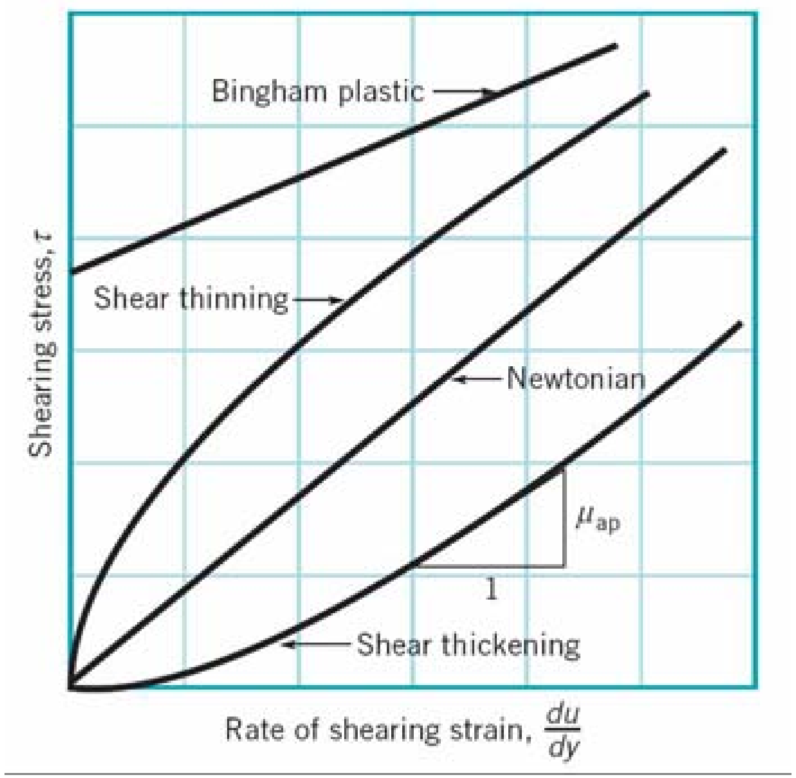
- Fluids for which the shearing stress is linearly related to the rate of shearing strain are designated as **Newtonian fluids** after I. Newton (1642-1727).
- Most common fluids such as water, air, and gasoline are Newtonian fluid under normal conditions.
- Fluids for which the shearing stress is not linearly related to the rate of shearing strain are designated as **non-Newtonian fluids.**

剪應力與剪應變率的關係呈線性者，為牛頓流體。否則為非牛頓流體。

Non-Newtonian Fluids 1/3

- ❖Shear thinning fluids.
- ❖Shear thickening fluids.
- ❖Bingham plastic

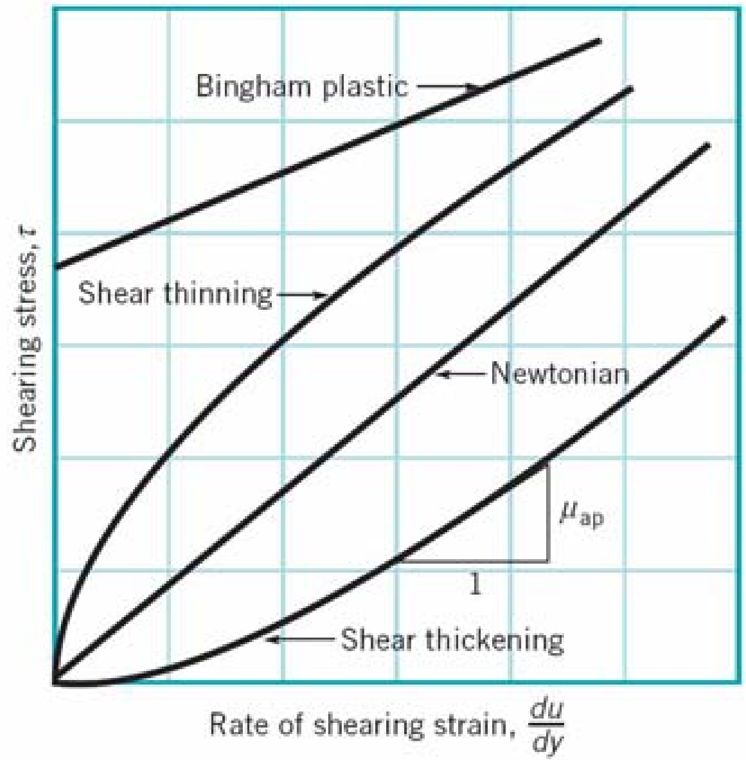
Shear stress與Rate of shearing strain間的關係



Non-Newtonian Fluids 2/3

剪力越大，黏度越低者，稱為
shear thinning fluids

- Shear thinning fluids: The viscosity decreases with increasing shear rate – the harder the fluid is sheared, the less viscous it becomes. Many colloidal suspensions and polymer solutions are shear thinning. Latex paint is example.



剪變薄：shear rate↑黏度↓
膠狀懸浮液與聚合物，如乳膠漆。

Non-Newtonian Fluids 3/3

剪力越大，黏度越高者，稱為 shear thickening fluids

- Shear thickening fluids: The viscosity increases with increasing shear rate – the harder the fluid is sheared, the more viscous it becomes. Water-corn starch mixture water-sand mixture are examples.

剪變厚：shear rate↑黏度↑
水與玉米澱粉混合物、水與沙混合物。

Non-Newtonian fluids

- Bingham plastic: neither a fluid nor a solid. Such material can withstand a finite shear stress without motion, but once the yield stress is exceeded it flows like a fluid. Toothpaste and mayonnaise are common examples.

Bingham plastic：當剪應力達到某一程度後，
物質才像流體者，如牙膏、蛋黃醬（美乃滋）。

Kinematic Viscosity

- Defining kinematic viscosity $\nu = \mu / \rho$ [Ny]
  - The dimensions of kinematic viscosity are L^2/T.
  - The units of kinematic viscosity in BG system are ft^2/s and SI system are **m^2/s**.
  - In the CGS system, the kinematic viscosity has the units of cm^2/s, is called a **stoke**, abbreviated St.

Compressibility of Fluids

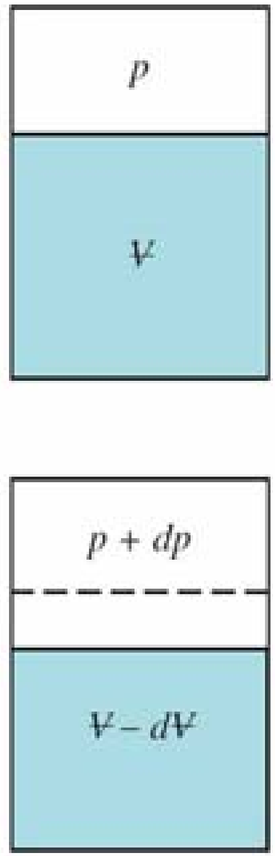
- Bulk modulus.
- Compression and expansion of gases.
- Speed of sound.

Bulk Modulus[1/2]

❖ How easily can the volume (and thus the density) of a given mass of the fluids be changed when there is a change in pressure? That is, how **compressible of the fluid ?**

❖ A property, **bulk modulus E_v**, is used to characterize compressibility of fluid.

定義 $E_v = -\dfrac{dp}{d\forall/\forall} = \dfrac{dp}{d\rho/\rho}$ 描述流體的壓縮性



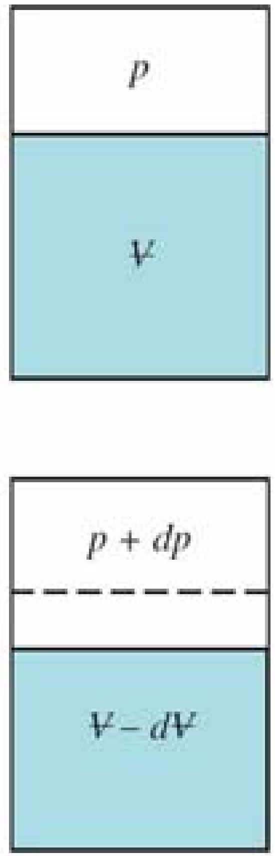
Bulk Modulus$^{1/2}$

$$E_v = -\frac{dp}{d\forall/\forall} = \frac{dp}{d\rho/\rho}$$

Is the differential change in pressure needed to create a differential change in volume.

與壓力一樣的因次

Bulk modulus越大，表示流體越不可壓縮。



❖ Liquids are usually considered to be incompressible, whereas gases are generally considered compressible.

液體可假設為不可壓縮，
氣體則必須考慮壓縮性。

Compression and Expansion

壓縮或膨脹過程，壓力與密度的關係…與過程的特性有關

❖ When gases are compressed or expanded, the relationship between pressure and density depends on the nature of the process.

等溫過程

☞ For isothermal process $\frac{p}{\rho} = \text{constant}$

$$E_v = -\frac{dp}{d\forall / \forall} = \frac{dp}{d\rho / \rho}$$

过程中壓力與密度的關係

>> $E_v = p$

等熵過程

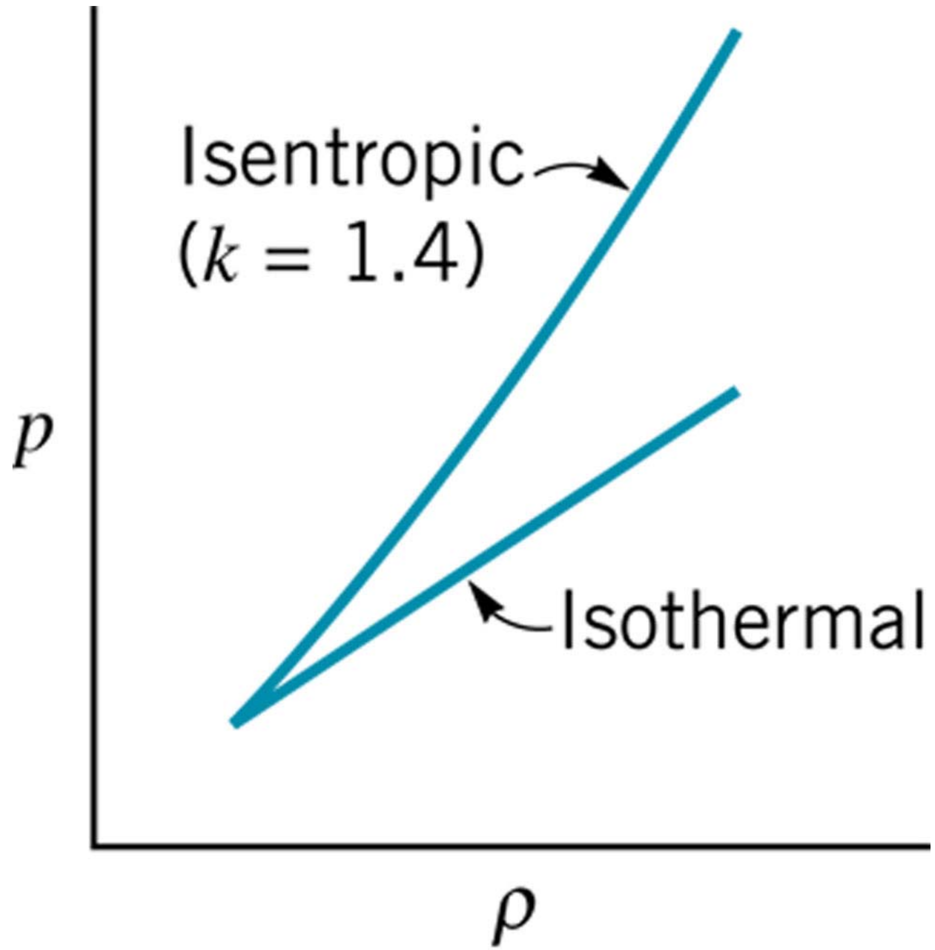
☞ For isentropic process $\frac{p}{\rho^k} = \text{constant}$

>> $E_v = k\,p$

Where k is the ratio of the specific heat at constant pressure, c_p, to the specific heat at constant volume, c_v.

$c_p - c_v = R$ = gas constant

P vs. ρ Depends on the type of process



Example 1.6 Isentropic Compression of a Gas

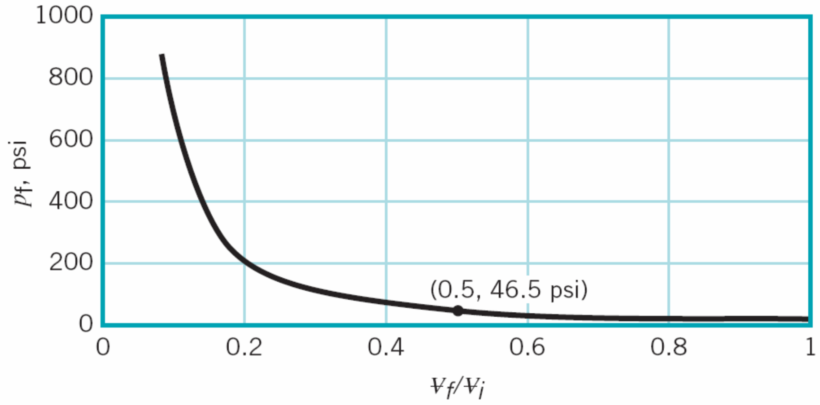
- A cubic foot of helium at an absolute pressure of 14.7 psi is compressed isentropically to ½ ft^3. What is the final pressure?

等熵過程壓力與密度的關係

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$

$$p_f = \left(\frac{\rho_f}{\rho_i}\right)^k p_i = (2)^{1.66}(14.7\text{psi})$$

Various value of the ration of the final volume to the initial volume



Speed of Sound 1/2

在流體中特定點給予一擾動後，該擾動在流體內散播的速度。

- Another important consequence of the compressibility of fluids is that disturbances introduced at some point in the fluid propagate at a finite velocity.
- For example, if a fluid is flowing in a pipe and a valve at the outlet is suddenly closed, the effect of the valve closure is not felt instantaneously upstream.
- It takes a finite time for the increased pressure created by the valve closure to propagate to an upstream location.

管的開口端閥門突然關掉，上游無法立即感受到閥門關閉的動作，但一段時間之後，因關閉閥門導致的上升壓力即傳播到上游。

Speed of Sound 2/2

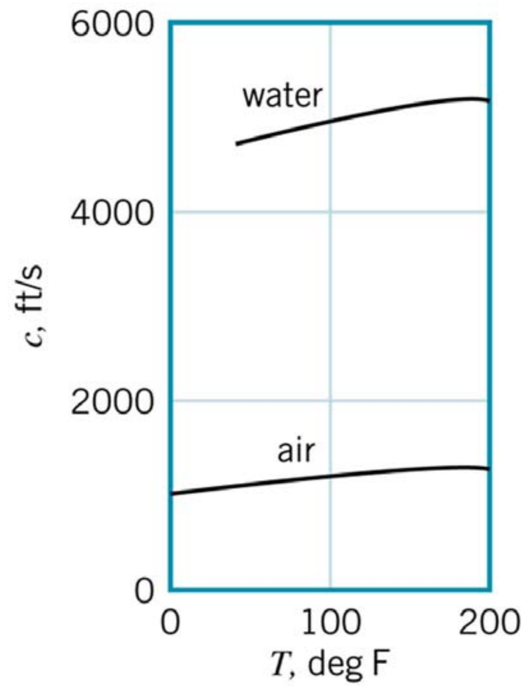
擾動在流體內散播的速度稱為聲速。

- The velocity at which small disturbances propagate in a fluid is called ***the speed of sound***. 與子彈一樣快速
- The speed of sound is related to change in pressure and density of the fluid medium through

聲速 $$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{E_v}{\rho}}$$

⇨ For isentropic process $c = \sqrt{\dfrac{kP}{\rho}}$

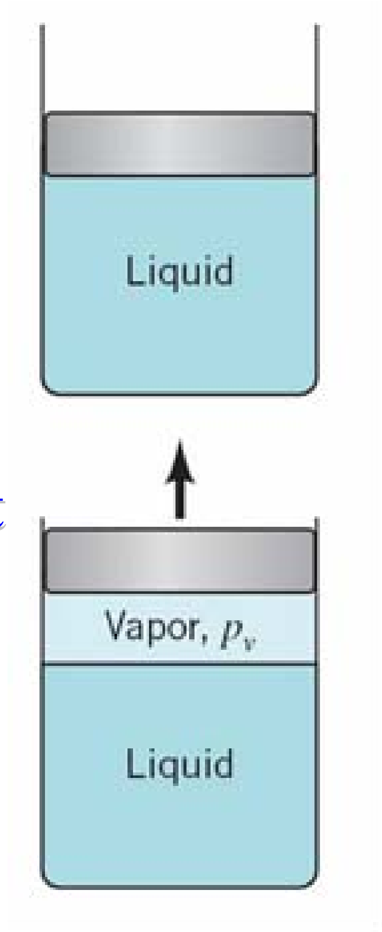
⇨ For ideal gas $c = \sqrt{kRT}$



Vapor Pressure and Boiling 1/2

容器內的液體直接與空氣接觸，一部份的液體分子克服分子間結合力，脫離液面進入大氣。

- ❖ If liquids are simply placed in a container open to the atmosphere, some liquid molecules will overcome the intermolecular cohesive forces and escape into the atmosphere.
- ❖ If **the container is closed** with small air space left above the surface, and this space evacuated to form a vacuum, a pressure will develop in the space as a result of the vapor that is formed by the escaping molecules.
- ❖ When an equilibrium condition is reached, the vapor is said to be saturated and the pressure that the vapor exerts on the liquid surface is termed the **VAPOR PRESSURE, p_v**.



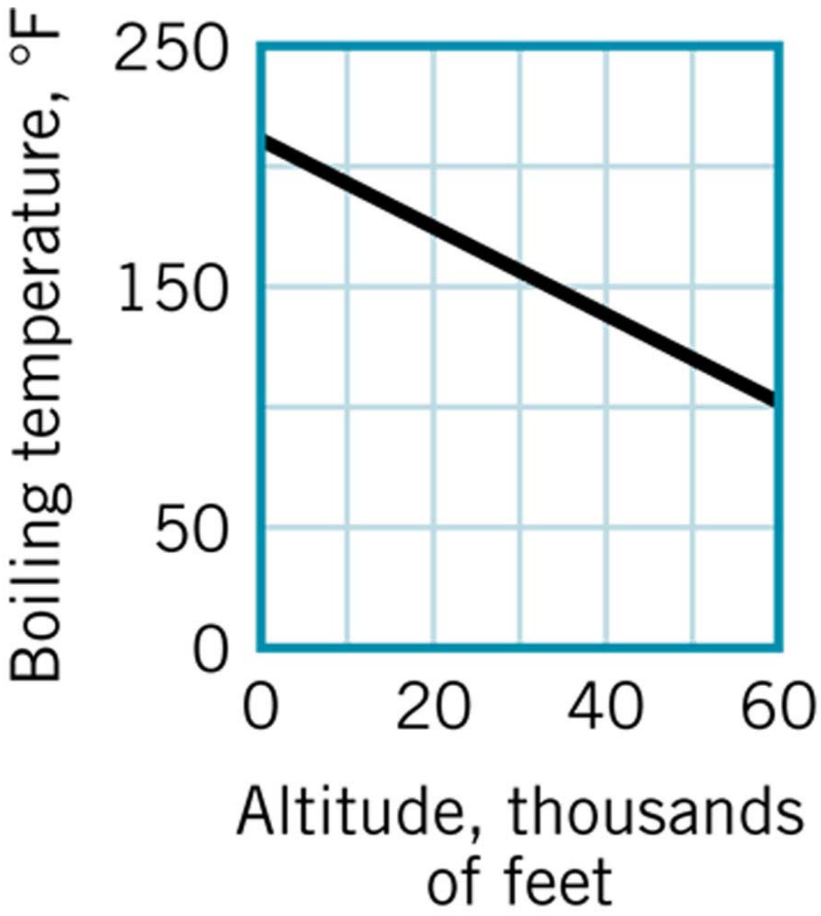
容器封閉，液面上預留小空間並抽成真空。當溢出的液體分子變成蒸氣，進入空間且達到平衡時，飽和的蒸氣施加在液面的壓力稱為vapor pressure。

Vapor Pressure and Boiling 2/2

- Vapor pressure is closely associated with molecular activity, the value of vapor pressure for a particular liquid depends on temperature. Vapor pressure與分子活動有關，也與溫度有關。
- Boiling, which is the formation of vapor bubbles within a fluid mass, is initiated when the absolute pressure in the fluid reaches the vapor pressure.
- The formation and subsequent collapse of vapor bubbles in a flowing fluid, called **cavitation**, is an important fluid flow phenomenon.

當流體內的壓力等於Vapor pressure時，流體內部出現vapor bubble，稱為boiling。
當流體內的vapor bubble形成而後破裂的現象稱為cavitation。空蝕是一項重要的流體現象。

Boiling temperature vs. Altitude



Surface Tension 1/3

- At the interface between a liquid and a gas, or between two immiscible liquids, forces develop in the liquid surface which cause the surface to behave as if it were a “skin” or “membrane” stretched over the fluid mass.
- Although such a skin is not actually present, this conceptual analogy allows us to explain several commonly observed phenomena.

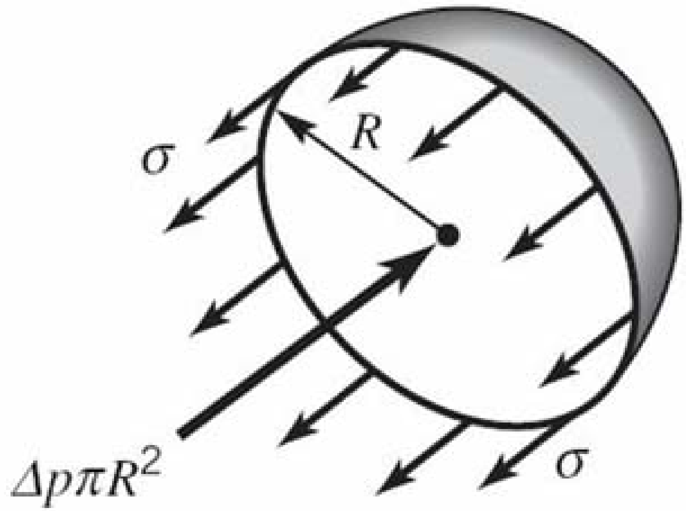
液體與氣體界面、互不相溶的液體間，在液體表面發展出來的力，讓液體表面像有一層皮膚或薄膜覆蓋在上面。這層皮膚或薄膜並不是真的存在，但這種想像力卻可用來解釋很多出現在液體表面的現象。

Floating razor blade

Surface Tension 2/3

沿著表面上任意一條線單位長度的分子吸引力強度，稱為surface tension

- Surface tension: the intensity of the molecular attraction per unit length along any line in the surface and is designated by the Greek symbol **σ**.



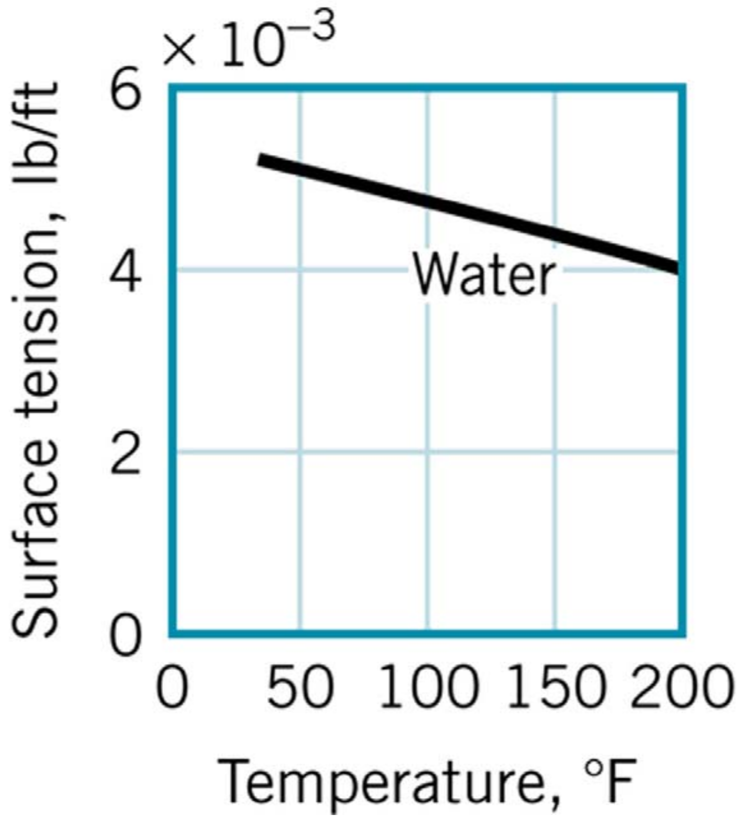
The force due to surface tension = **The force due to pressure difference**

$$2\pi R\sigma = \Delta p \pi R^2$$

$$\Delta p = p_i - p_e = \frac{2\sigma}{R}$$

Where p_i is the internal pressure and p_e is the external pressure

Surface tension vs. Temperature



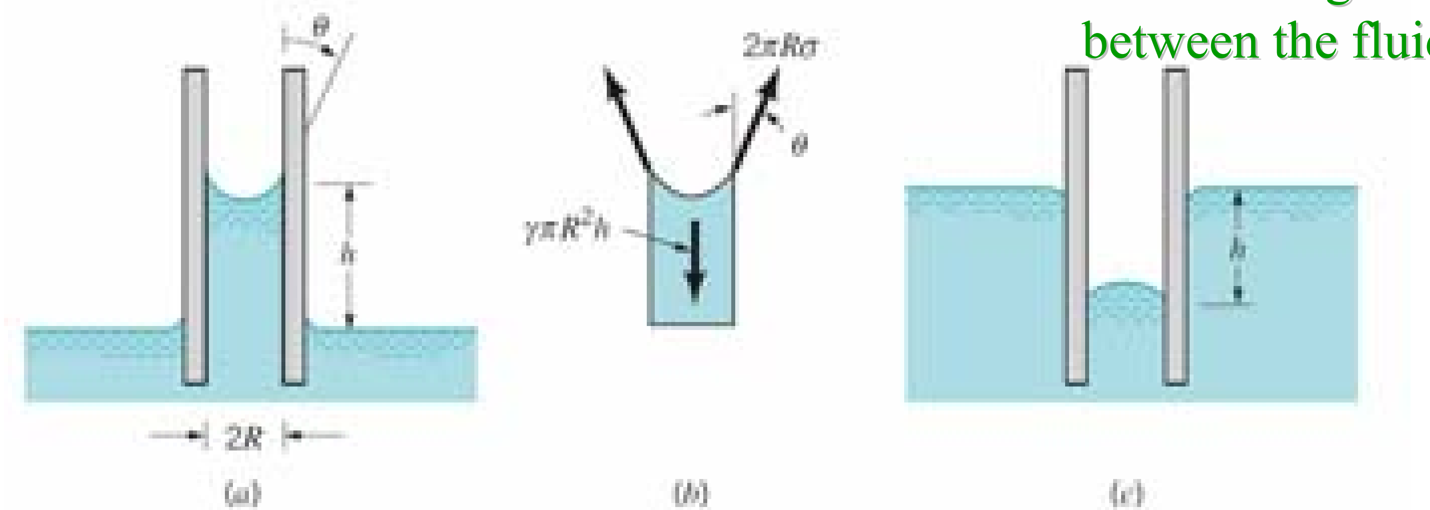
Surface Tension 3/3

Capillary rise

常觀察到的現象：毛細管。

❖ A common phenomena associated with surface tension is the rise or fall of a liquid in a capillary tube.

Θ is the angle of contact between the fluid and tube.

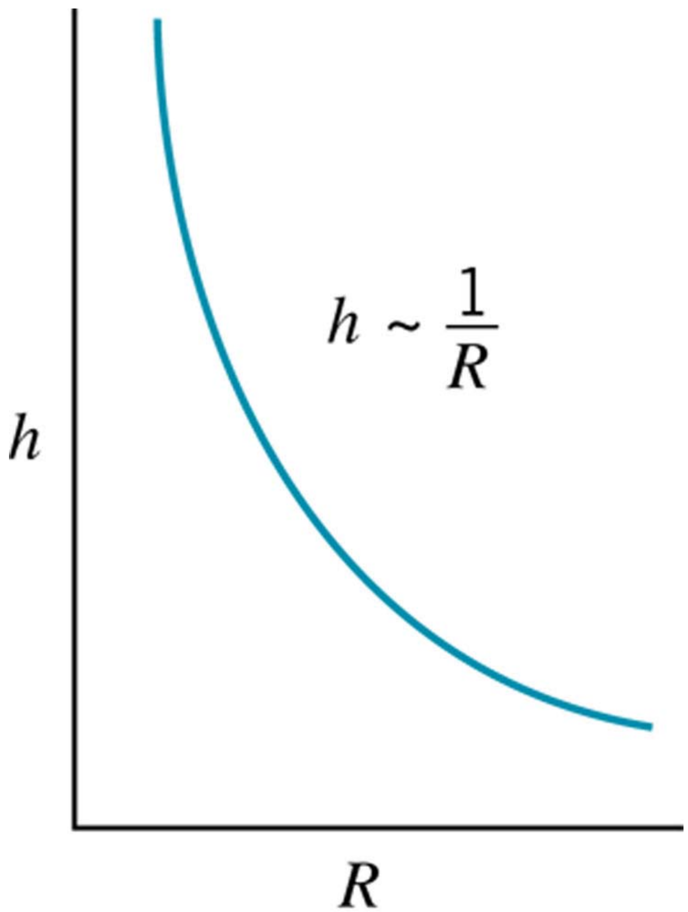


Balance for equilibrium

$$\gamma\pi R^2 h = 2\pi R\sigma\cos\theta$$

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

Height h vs. Tube radius R

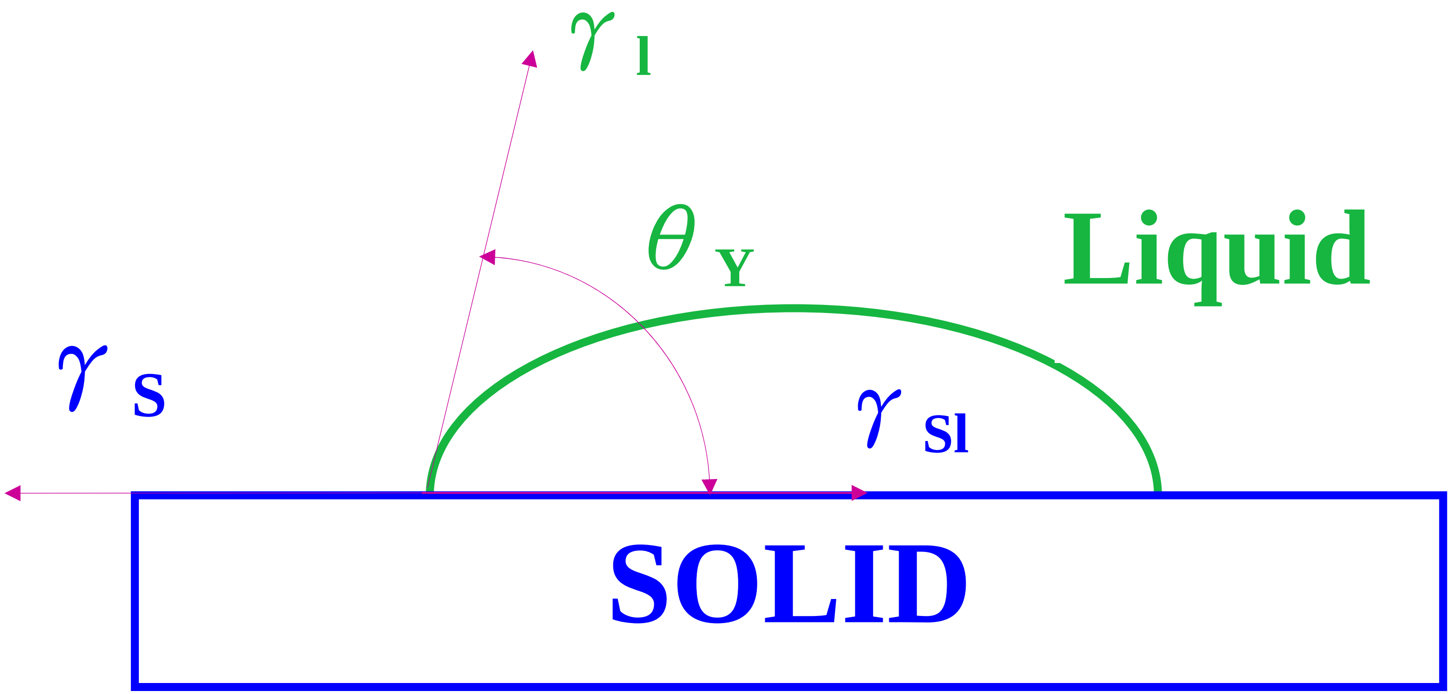


Surface Tension Effects

- Surface tension effects play a role in many fluid mechanics problems including the movement of liquids through soil and other porous media, flow of thin film, formation of drops and bubbles, and the breakup of liquid jets.
- Surface phenomena associated with liquid-gas, liquid-liquid or liquid-gas-solid interfaces are exceedingly complex.

表面張力在流體力學中扮演重要的角色：土壤內或孔隙介質內液體的流動、薄膜的流動，水滴或水泡的形成。
表面現象：liquid-gas、liquid-liquid、 liquid-gas-solid。

Measurement of Surface Tension 1/2



Measurement of Surface Tension 2/2