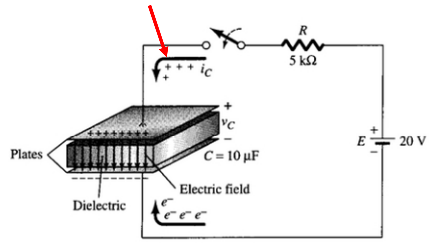
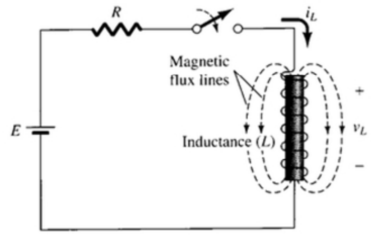
Charging & Discharging 1/3

電流是變動的，不再用大寫的 I



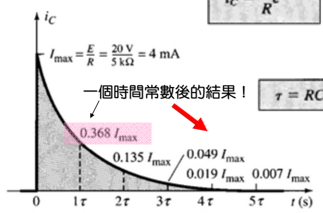
Charging & Discharging 1/3



Charging & Discharging 2/3

$$i_C = \frac{E}{R} e^{-t/\tau}$$

電流→0

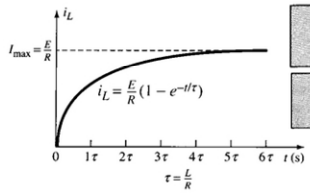


一個時間常數後的結果！ $\tau = RC$

i_c由E/R降低到零；V_c由零充電上升到E時，電容器如同open circuit！

Charging & Discharging 2/3

- 流經coil的電流i_L，與電容器的v_c相似。電流i_L由零遞增到E/R。

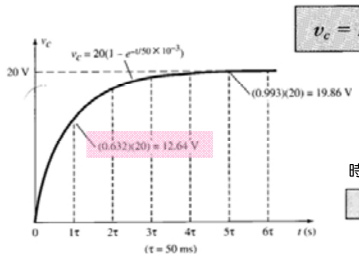


$$i_L = \frac{E}{R}(1 - e^{-t/\tau})$$

$$\tau = \frac{L}{R}$$ (seconds)

Charging & Discharging 3/3

$$v_c = E(1 - e^{-t/\tau})$$

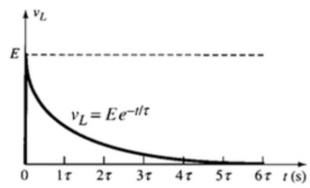


電壓0→

時間常數 $\tau = RC$

Charging & Discharging 3/3

- 跨越coil的電壓v_L，與電容器的i_c相似。電壓v_L由E降低到零。



$$V_L = Ee^{-t/\tau}$$

$$\tau = \frac{L}{R}$$ (seconds)

v- i Relationship 1/2

前面講的電壓是DC，一旦外加電壓是變動時…

- If the external voltage applied to the capacitor plates changes in time:

$$q(t) = cv(t)$$ A time-varying voltage will cause charge to vary in time.

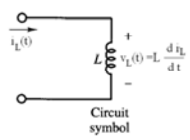
- Recalling the definition of current:

$$i(t) = \frac{dq(t)}{dt} \Rightarrow i(t) = C\frac{dv(t)}{dt}$$ which is called **CIRCUIT LAW for a CAPACITOR.**

v- i Relationship 1/2

- 電感的電壓大小為電感量與電流變動率的乘積，此為電感的『歐姆定律』：

$$v_L(t) = L\frac{di_L}{dt}$$



L is called the inductance of the coil（電感）；
電感的單位 H = V s/A ：
v_L=感應電壓；L = 電感量；di_L/dt=電流變動率

v- i Relationship 2/2

- The voltage across a capacitor:

$$v_C(t) = \frac{1}{C}\int_{\infty}^{t} i_C(t')dt'$$

- The capacitor voltage depends on the past current through the capacitor, up until the present time, t.

$$v_o = v_C(t = t_o) = \frac{1}{C}\int_{\infty}^{t_o} i_C(t')dt'$$

$$v_C(t) = \frac{1}{C}\int_{t_o}^{t} i_C(t')dt' + v_o \quad t \geq t_o$$

where v_o is sufficient to account **for the entire past history** of the capacitor current.

v- i Relationship 2/2

- The current through a inductor:

$$i_L(t) = \frac{1}{L}\int_{\infty}^{t} v_L(t')dt'$$

- The inductor current depends on the past voltage across the inductor, up until the present time, t.

$$I_o = i_L(t = t_o) = \frac{1}{L}\int_{\infty}^{t_o} v_L(t')dt'$$

$$i_L(t) = \frac{1}{L}\int_{t_o}^{t} v_L(t')dt' + i_o \quad t \geq t_o$$

where i_o is sufficient to account for the entire past history of the inductor voltage.

Capacitor $v_c \rightarrow i_c$

$$i_c = C\frac{dv_c}{dt} \approx C\frac{\Delta v_c}{\Delta t}$$

$0 \rightarrow 2ms$

$$i_c = (2\mu F)\frac{8V}{2ms} = +8mA$$

$2 \rightarrow 5ms$

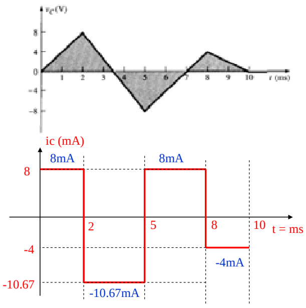
$$i_c = (2\mu F)\frac{-16V}{3ms} = -10.67mA$$

$5 \rightarrow 8ms$

$$i_c = (2\mu F)\frac{12V}{3ms} = +8mA$$

$8 \rightarrow 10ms$

$$i_c = (2\mu F)\frac{-4V}{2ms} = -4mA$$



Inductor $i_L \rightarrow v_L$

$$v_L = L\frac{di_L}{dt} \approx L\frac{\Delta i_L}{\Delta t}$$

$0 \rightarrow 3ms$

$$v_L = 0V$$

$3 \rightarrow 7ms$

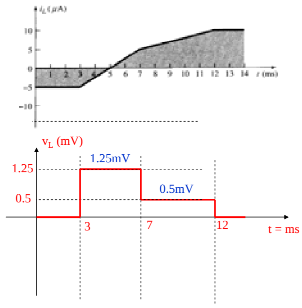
$$v_L = (0.5H)\frac{10\mu A}{4ms} = 1.25mV$$

$7 \rightarrow 12ms$

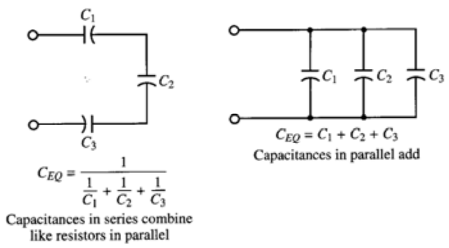
$$v_L = (0.5H)\frac{5\mu A}{5ms} = 0.5mV$$

$12 \rightarrow$

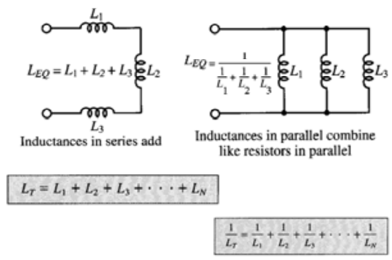
$$v_L = 0V$$



電容器串、並聯



電感串、並聯



Energy Storage in Capacitor

- The energy stored in the capacitor, Wc（t）

$$W_C(t) = \int P_C(t')dt' = \int v_C(t')i_C(t')dt' = \int v_C(t')C\frac{dv_C(t')}{dt'}dt'$$

$$W_C(t) = \frac{1}{2}Cv_C^2(t)$$

Energy Storage in Inductor

- The energy stored in the inductor, W_L（t）

$$P_L(t) = i_L(t)v_L(t) = i_L(t)L\frac{di_L(t)}{dt} = \frac{d}{dt}\left[\frac{1}{2}Li_L^2(t)\right]$$

$$W_L(t) = \int P_L(t')dt' = \int v_L(t')i_L(t')dt' = \int i_L(t')L\frac{di_L(t')}{dt'}dt'$$

$$W_L(t) = \frac{1}{2}Li_L^2(t)$$

電容的正弦響應 1/2

$i_C(t) = C\frac{dv_C(t)}{dt}$　　$v_L(t) = L\frac{di_L}{dt}$

- 假設有一正弦電流通過一個電容，則電容的電壓響應方程式為？

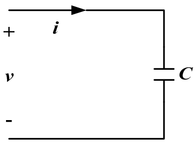
$$i = C\frac{dv}{dt} \qquad v = V_{peak}\sin\omega t$$

$$= C\frac{d}{dt}V_{peak}\sin\omega t$$

$$= \omega C V_{peak}\cos\omega t$$

$$= I_{peak}\cos\omega t$$

$$= I_{peak}\sin(\omega t + 90^\circ)$$



$$I_{peak} = \omega C V_{peak} = \frac{V_{peak}}{X_c}$$

$$X_c = \frac{1}{2\pi f C}$$

50

電感的正弦響應 1/2

$i_C(t) = C\frac{dv_C(t)}{dt}$　　$v_L(t) = L\frac{di_L}{dt}$

- 假設有一正弦電流通過一個電感，則電感的電壓響應方程式為？

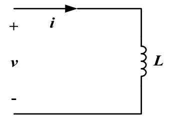
$$v = L\frac{di}{dt} \qquad i = I_{peak}\sin\omega t$$

$$= L\frac{d}{dt}I_{peak}\sin\omega t$$

$$= \omega L I_{peak}\cos\omega t$$

$$= V_{peak}\cos\omega t$$

$$= V_{peak}\sin(\omega t + 90^\circ)$$



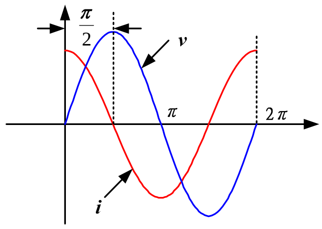
$$V_{peak} = \omega L I_{peak} = X_L I_{peak}$$

$$X_L = \omega L = 2\pi f L$$

44

電容的正弦響應 2/2

修正

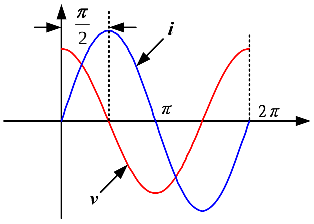


電壓與電流間有相位差，流經電感的電流，比跨越電感的電壓，領先90°。或者說，電壓落後電流90°。

51

電感的正弦響應 2/2

修正



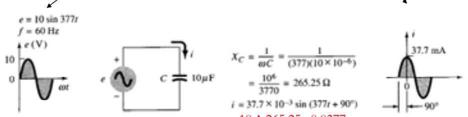
電壓與電流間有相位差，流經電感的電流，比跨越電感的電壓，落後90°。或者說，電壓領先電流90°。

45

Capacitor C 1/3

- 當sinusoidal voltage跨越電容時，電壓、電流與電容的關係如何？

電壓與電流間有相位差，流經電感的電流，比跨越電感的電壓，領先90°。或者說，電壓落後電流90°。



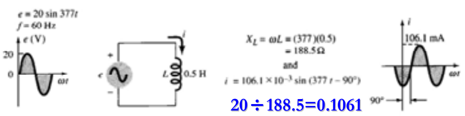
$10 \div 265.25 = 0.0377$

52

Inductor L 1/3

- 當sinusoidal voltage跨越電感時，電壓、電流與電感的關係如何？

電壓與電流間有相位差，流經電感的電流，比跨越電感的電壓，落後90°。或者說，電壓領先電流90°。



$20 \div 188.5 = 0.1061$

46

Capacitor C 2/3

電壓的peak value與電流的peak value關係

$$I_{peak} = \frac{V_{peak}}{X_C}$$

其中，X_C稱為電容的電抗（reactance），單位為Ohms。電容的電抗就像電阻（器）的電阻一樣，用來限制電流的流動。

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \text{(ohms)}$$

53

Inductor L 2/3

電壓的peak value與電流的peak value關係

$$I_{peak} = \frac{V_{peak}}{X_L}$$

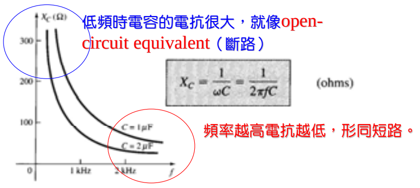
其中，X_L稱為電感的電抗（reactance），單位為Ohms。電感的電抗就像電阻（器）的電阻一樣，用來限制電流的流動。

$$X_L = \omega L = 2\pi f L$$

47

Capacitor C 3/3

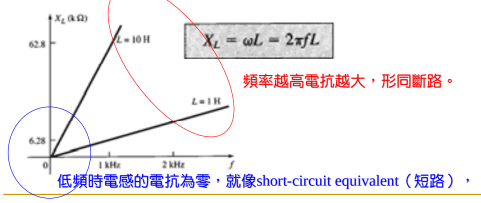
❑ 電容對於AC signal的反應與電阻不一樣。電容不像電阻消耗功率，而是將電能分別以電場的型態儲存起來，且它們的reactance（電抗）與頻率有關。



54

Inductor L 3/3

❑ 電感對於AC signal的反應與電阻不一樣。電感不像電阻消耗功率，而是將電能分別以磁場的型態儲存起來，且它們的reactance（電抗）與頻率有關。

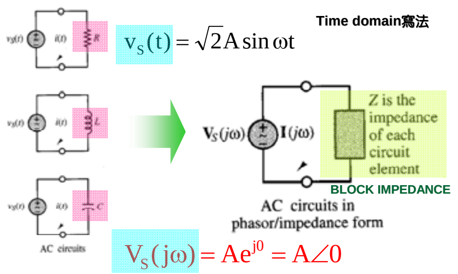


48

Generalized Impedance

Time domain寫法

$v_S(t) = \sqrt{2}A\sin\omega t$



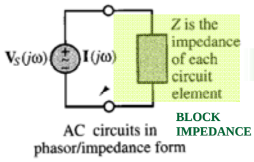
$V_S(j\omega) = Ae^{j0} = A\angle 0$

$v(t) = \sqrt{2}A\sin(wt+\phi) \Leftrightarrow V(jw) = Ae^{j\phi} = A\angle\phi$

81

Generalized Impedance

$v_S(t) = \sqrt{2}A\sin\omega t$ ⇒ $V_S(j\omega) = Ae^{j0} = A\angle 0$



$$Z(j\omega) = \frac{V_S(j\omega)}{I(j\omega)}$$

$v(t) = \sqrt{2}A\sin(wt+\phi) \Leftrightarrow V(jw) = Ae^{j\phi} = A\angle\phi$

82

Capacitor 1/3

❑ Recalling the defining relationships for the ideal capacitor：

$$i_C(t) = C\frac{dv_C(t)}{dt} \qquad v_C(t) = \frac{1}{C}\int i_C(t')dt'$$

❑ Let $v_C(t)= v_S(t)$ and $i_C(t) = i_S(t)$， then the following expression may be derived for the capacitor current：

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}(\sqrt{2}A\sin\omega t)$$

$$= C(\sqrt{2}A\omega\cos\omega t) = \omega C\sqrt{2}A\sin(\omega t + \frac{\pi}{2})$$

$v(t) = \sqrt{2}A\sin(wt+\phi) \Leftrightarrow V(jw) = Ae^{j\phi} = A\angle\phi$

87

Inductor 1/3

❑ Recalling the defining relationships for the ideal inductor：

$$v_L(t) = L\frac{di_L(t)}{dt} \qquad i_L(t) = \frac{1}{L}\int v_L(t')dt'$$

❑ Let $v_L(t)= v_S(t)$ and $i_L(t) = i_S(t)$， then the following expression may be derived for the inductor current：

$$i_L(t) = \frac{1}{L}\int v_S(t')dt' = -\frac{\sqrt{2}A}{\omega L}\cos\omega t = \frac{1}{L}\int \sqrt{2}A\sin\omega t' dt'$$

$$= -\frac{\sqrt{2}A}{\omega L}\cos\omega t = \frac{\sqrt{2}A}{\omega t}\sin(\omega t - \frac{\pi}{2})$$

$v(t) = \sqrt{2}A\sin(wt+\phi) \Leftrightarrow V(jw) = Ae^{j\phi} = A\angle\phi$

84

Capacitor 2/3

❑ This result can be seen by writing the capacitor voltage and current in time-domain form：

$$v_C(t) = v_S(t) = \sqrt{2}A\sin(\omega t)$$

$$i(t) = i_C(t) = \omega C\sqrt{2}A\sin(\omega t + \frac{\pi}{2})$$

The capacitor current is shifted in phase by 90 with respect to the voltage.（電容電流與電壓的相位移為領先90˚）

❑ 電容電流的大小不僅是電壓大小的scaled version而已，而是還depends on the frequency ω，甚至與電壓有相位落後！因為電容不是單純的電阻！

$v(t) = \sqrt{2}A\sin(wt+\phi) \Leftrightarrow V(jw) = Ae^{j\phi} = A\angle\phi$

88

Inductor 2/3

❑ This result can be seen by writing the inductor voltage and current in time-domain form：

$$v_L(t) = v_S(t) = \sqrt{2}A\sin(\omega t)$$

$$i(t) = i_L(t) = \frac{\sqrt{2}A}{\omega t}\sin(\omega t - \frac{\pi}{2})$$

The inductor current is shifted in phase by 90 with respect to the voltage.（電感電流與電壓的相位移為落後90˚）

❑ 電感電流的大小不僅是電壓大小的scaled version而已，而是還depends on the frequency ω，甚至與電壓有相位落後！因為電感不是單純的電阻！

$v(t) = \sqrt{2}A\sin(wt+\phi) \Leftrightarrow V(jw) = Ae^{j\phi} = A\angle\phi$

85

Capacitor 3/3

- Using phasor notation：

$$V_S(j\omega) = A\angle 0 \qquad I(j\omega) = \omega CA\angle \pi/2$$

- **The impedance of the capacitor is defined as**

$$Z_C(j\omega) = \frac{V_S(j\omega)}{I(j\omega)} = \frac{1}{\omega C}\angle -\pi/2 = \frac{1}{j\omega C}$$

The impedance of the capacitor varying as an inverse function of frequency。在低頻時，電容的impedance很高，就像開路一般；相對的，在高頻時，電容的impedance很低，就像短路一般。

$$v(t) = \sqrt{2}A\sin(wt+\phi) \Leftrightarrow V(jw) = Ae^{j\phi} = A\angle\phi$$

89

Inductor 3/3

- Using phasor notation：

$$V_S(j\omega) = A\angle 0 \qquad I(j\omega) = \frac{A}{\omega L}\angle -\pi/2$$

- **The impedance of the inductor is defined as**

Impedance與頻率成正比

$$Z_L(j\omega) = \frac{V_S(j\omega)}{I(j\omega)} = \omega L\angle \pi/2 = j\omega L$$

An inductor will IMPEDE current flow in proportional to the sinusoidal frequency of the source signal。在低頻時，電感的impedance很低，就像短路一般；相對的，在高頻時，電感的impedance很高，就像開路一般。

$$v(t) = \sqrt{2}A\sin(wt+\phi) \Leftrightarrow V(jw) = Ae^{j\phi} = A\angle\phi$$

86