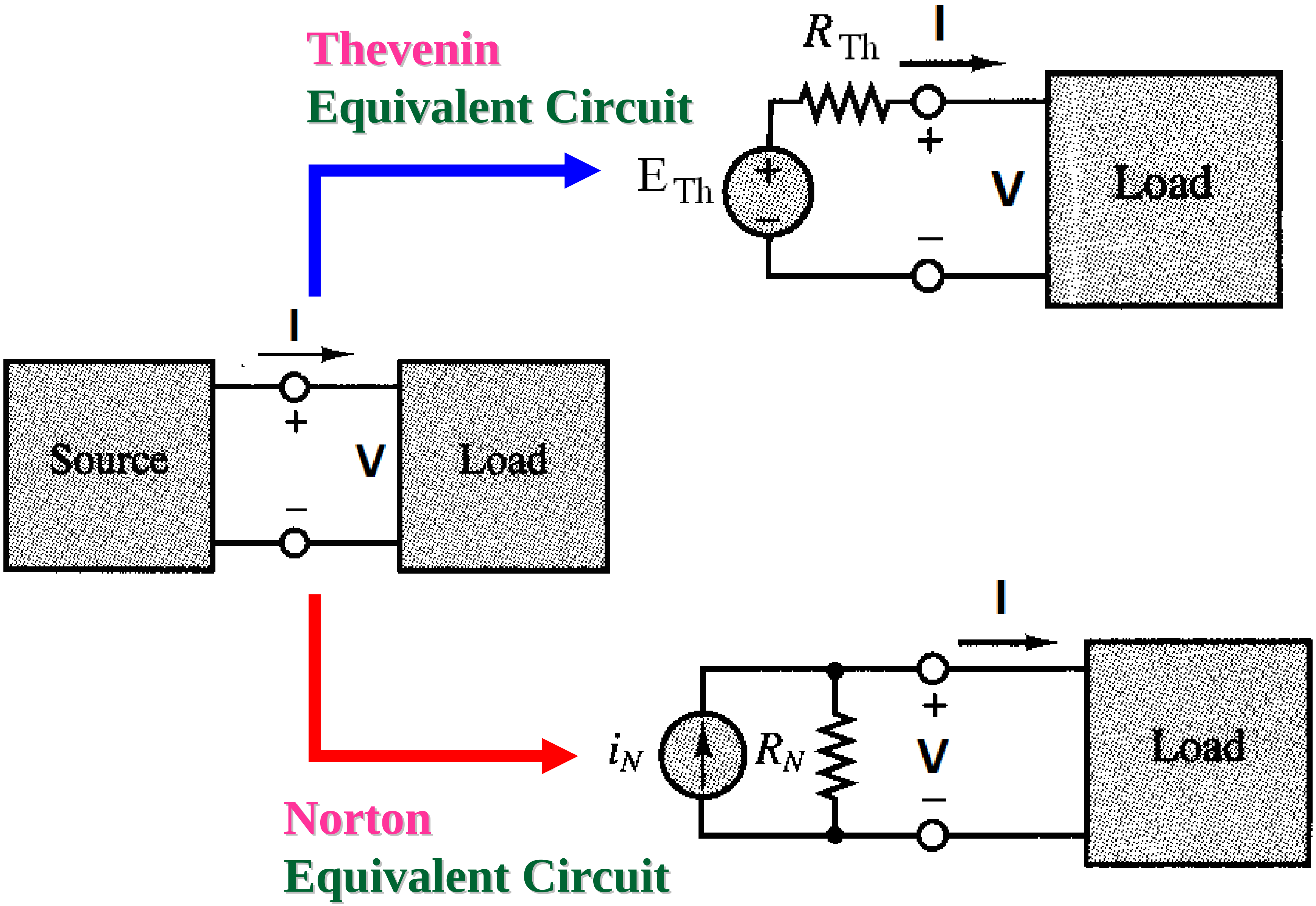
Thevenin vs. Norton

謝志誠

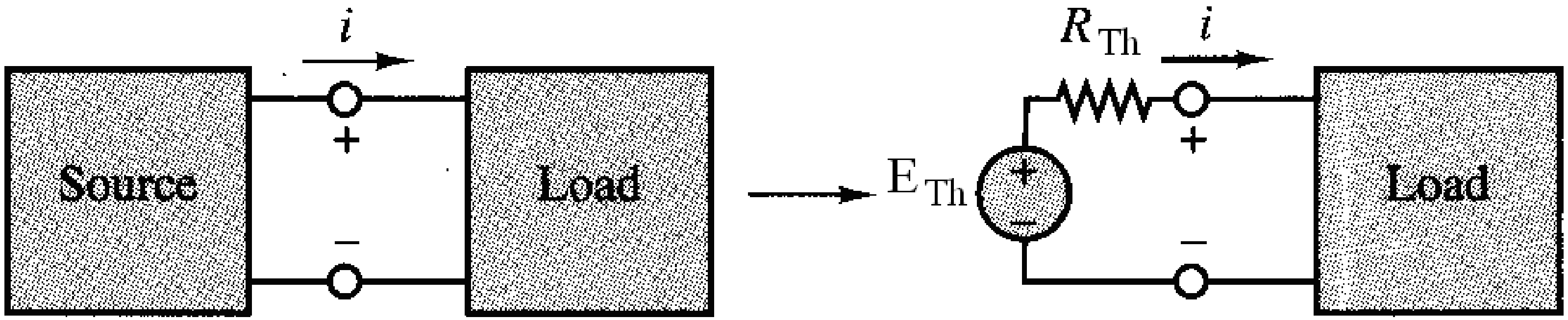
最新更新：2012年4月11日



Thevenin
Equivalent Circuit
R_{Th}
I
E_{Th}
+
−
V
Load
I
+
Source
V
Load
−
I
i_N
R_N
+
V
−
Load
Norton
Equivalent Circuit

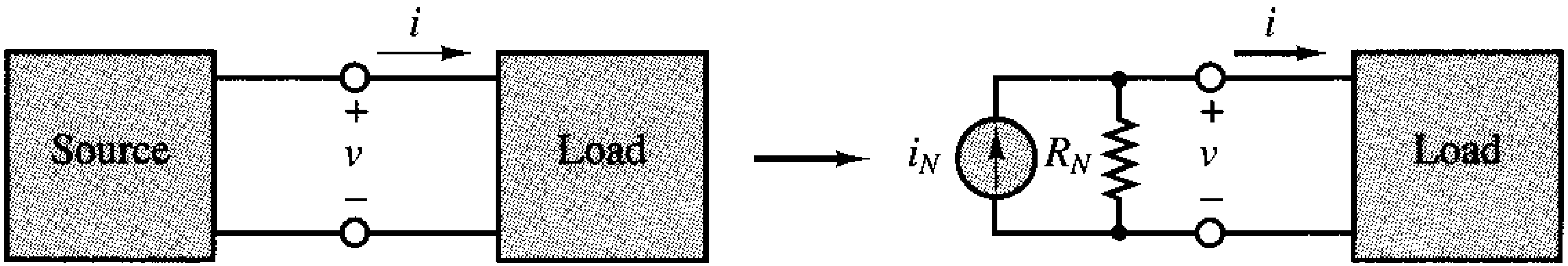
Thevenin theorem

- 戴維寧定理又稱等效電壓源定律，是由法國科學家Thevenin於1883年提出的一個電學定理。
- 一個接於兩點間，含有獨立電壓源、獨立電流源及電阻的線性電路，就其外部型態而言，在電性上可以用一個獨立電壓源E_{Th}（戴維寧等效電壓）和一個串聯的電阻R_{Th}（戴維寧等效電阻）組合來等效。



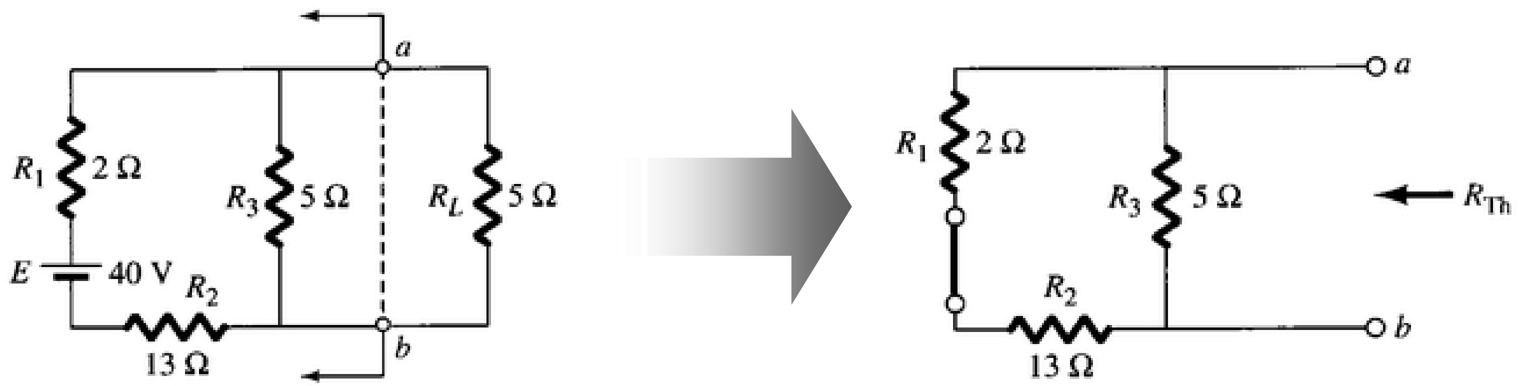
Norton theorem

- 諾頓定理是戴維寧定理的一個延伸，於1926年由Hause-Siemens研究員漢斯·費迪南·梅耶爾及貝爾實驗室工程師愛德華·羅里·諾頓分別提出。
- 一個接於兩點間，含有獨立電壓源、獨立電流源及電阻的線性電路，就其外部型態而言，在電性上可以用一個獨立電流源I_n（諾頓等效電流）和一個並聯的電阻R_N（戴維寧等效電阻）組合來等效。



R_{Th}

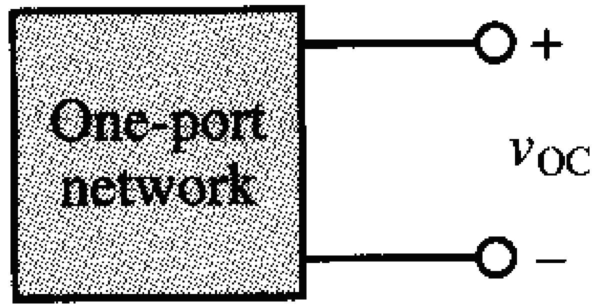
- ❑ 將欲探討的支路上的元件（負載）移走，成為開路。
- ❑ 將電路中所有的電壓源短路，電流源開路，求元件移走後電路兩端的等效電阻R_{Th}。

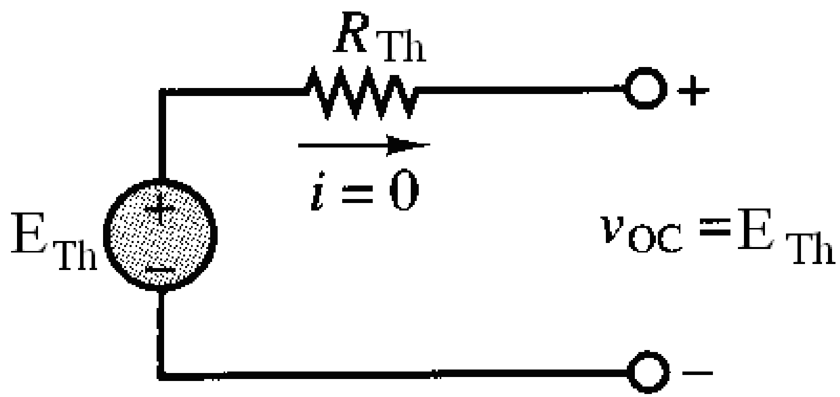


$$R_{Th} = R_3 \| (R_1 + R_2) = \frac{(5\ \Omega)(15\ \Omega)}{5\ \Omega + 15\ \Omega} = \mathbf{3.75\ \Omega}$$

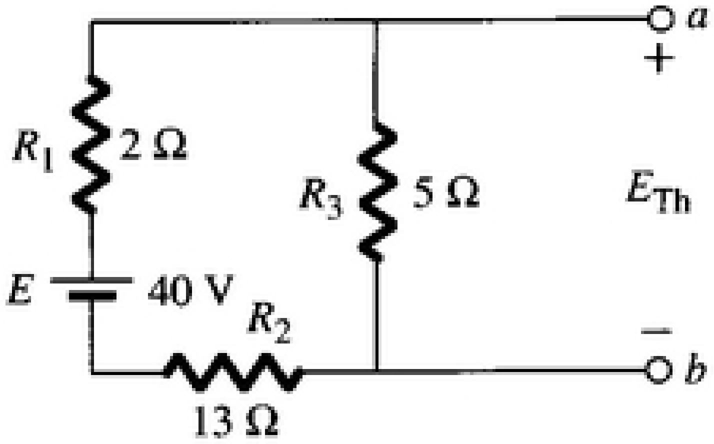
E_{Th} 1/2

- 將電壓源、電流源置回原處，再應用各種解電路的方法求出元件移走後，電路兩端的電壓，即為等效電壓E_{Th}。





E_{Th} 2/2

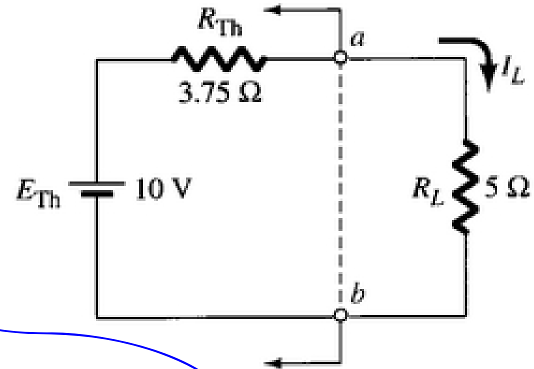


$$E_{Th} = V_{R_3} = \frac{R_3 E}{R_T} = \frac{(5\ \Omega)(40\ \text{V})}{20\ \Omega}$$

$$= \frac{200\ \text{V}}{20} = \mathbf{10\ V}$$

即使改變負載，也很容易計算改變後的電流多！

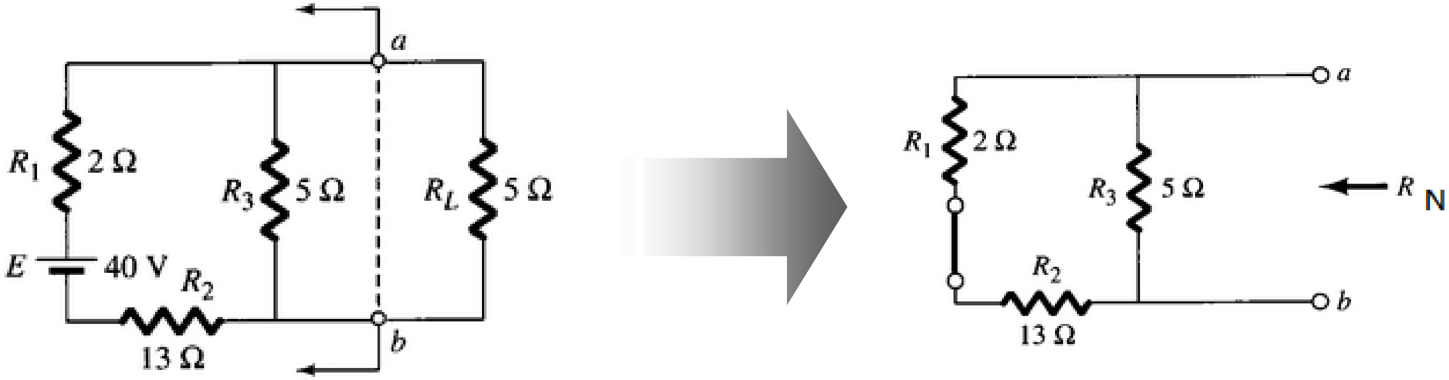
$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{10\ \text{V}}{3.75\ \Omega + 30\ \Omega} = \mathbf{0.296\ A}$$



$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{10\ \text{V}}{3.75\ \Omega + 5\ \Omega} = \mathbf{1.143\ A}$$

$R_N = R_{Th}$

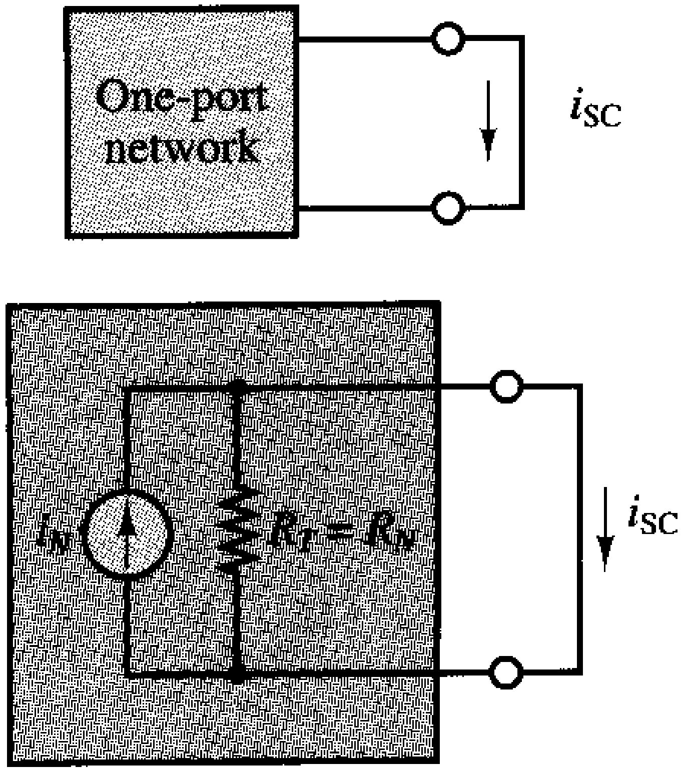
- 將欲探討的支路上的元件（負載）移走，成為開路。
- 將電路中所有的**電壓源短路，電流源開路**，求元件移走後電路兩端的等效電阻R_N。



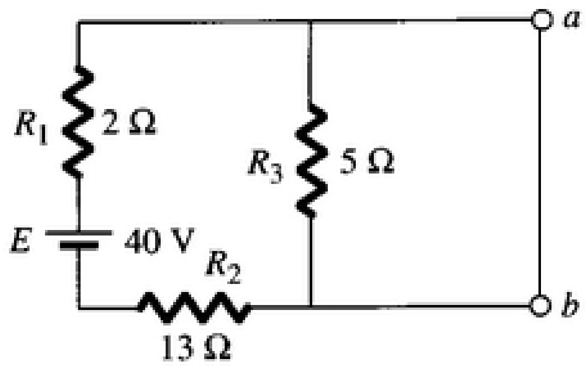
$$R_N = (R_1 + R_2) // R_3 = 3.75\Omega$$

i_N 1/2

- 將電壓源及電流源放回去，且將移去的負載元件兩端設定為短路，再應用前述各種求解電路的方法，求出流過此短路線上的電流i_{SC}，此電流即為等效電流i_N。

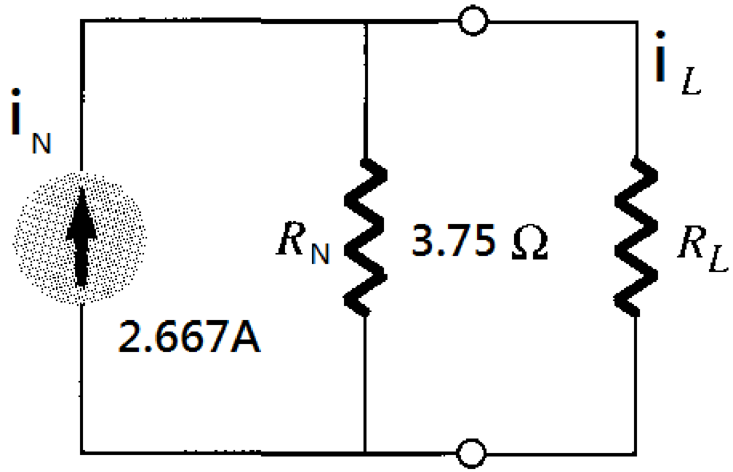


i_N 2/2



i_N

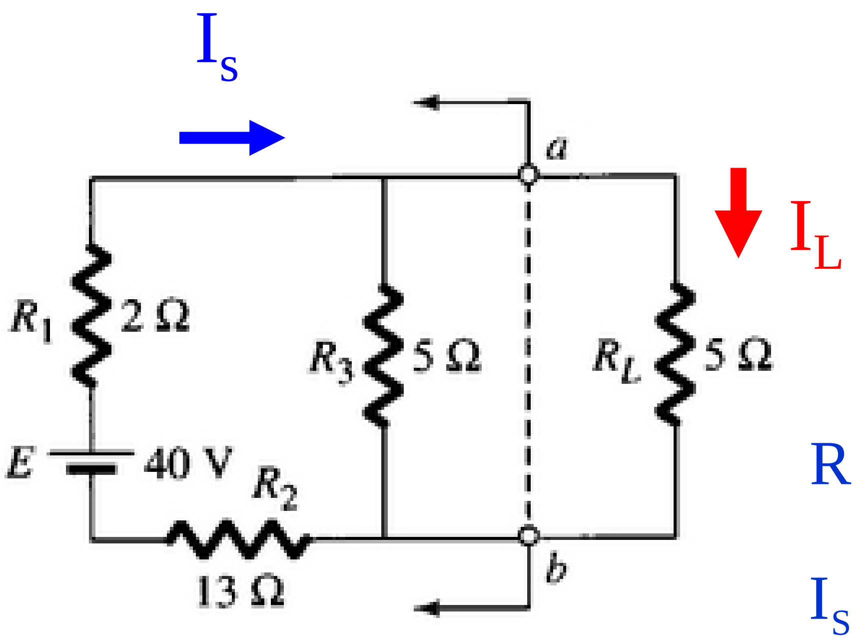
$$i_N = \frac{E}{R_1+R_2} = \frac{40V}{15\Omega} = 2.667A$$



$$i_L = i_N \times \frac{R_N}{R_N+R_L} = 1.143A$$

General Method

驗證



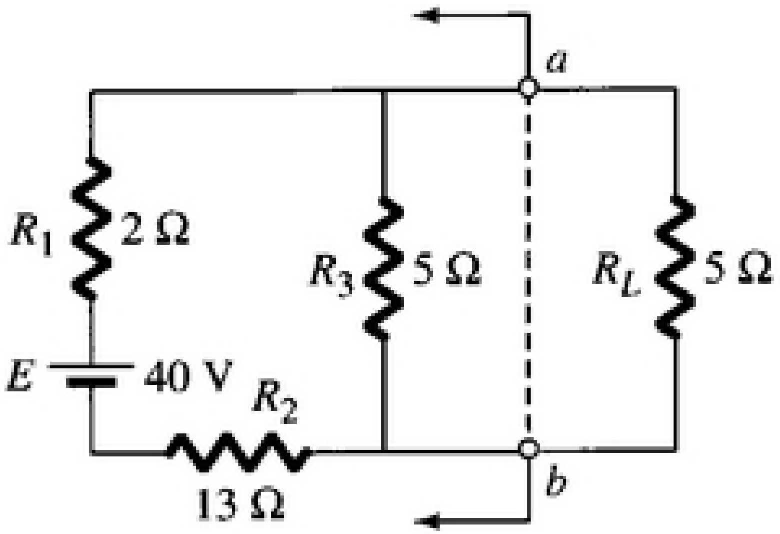
$$R_T = (R_L // R_3) + R_1 + R_2 = 17.5\Omega$$

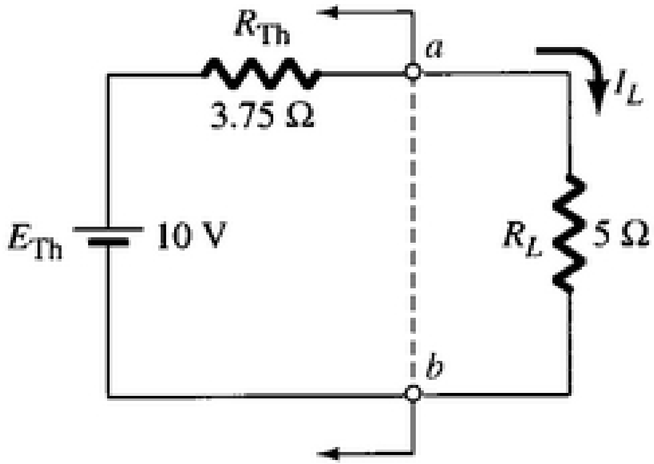
$$I_S = \frac{E}{R_T} = 2.286A$$

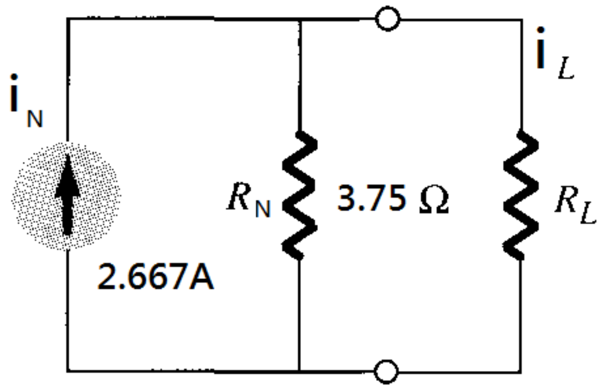
$$I_L = I_S \times \frac{R_3}{R_3 + R_L} = 1.143A$$

Source Conversion

驗證



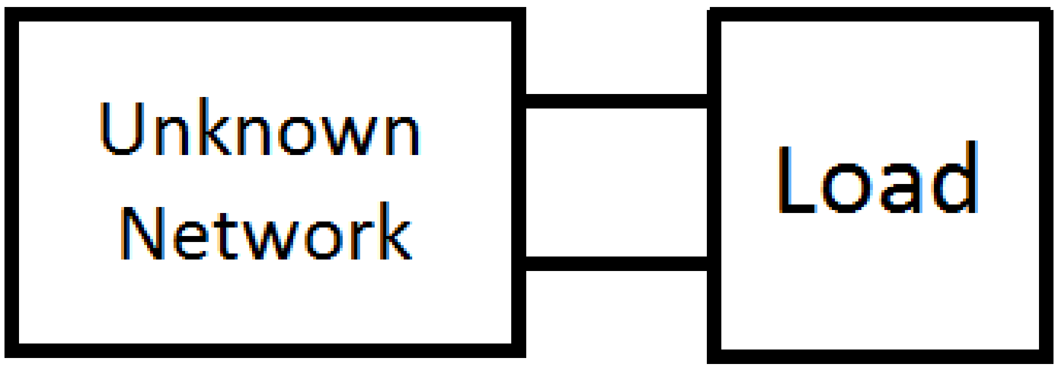




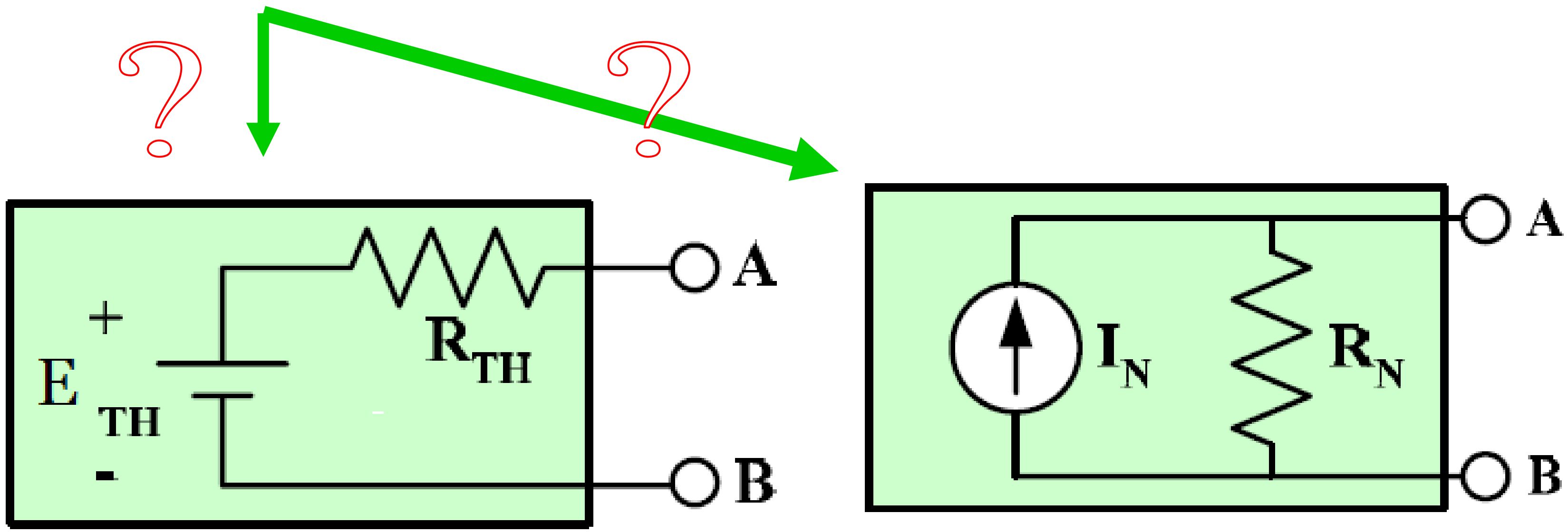
$R_N = R_{Th}$

$$i_N = \frac{E_{Th}}{R_{Th}} \qquad E_{Th} = i_N \times R_N$$

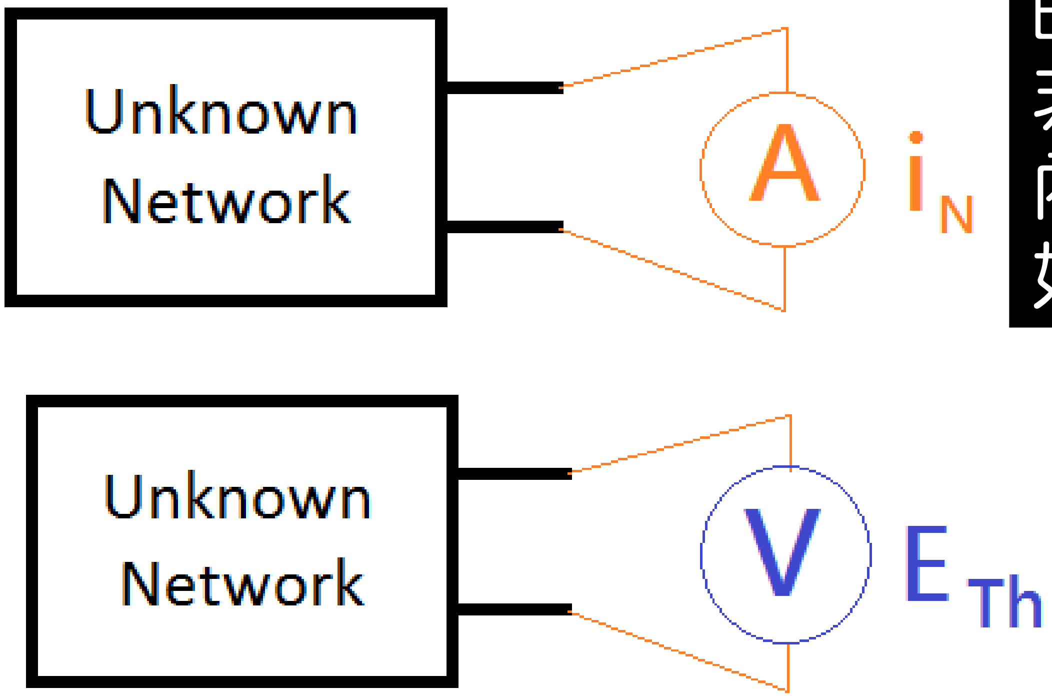
Application 1/2



把未知電路表
達成Thevenin
與Norton等效？



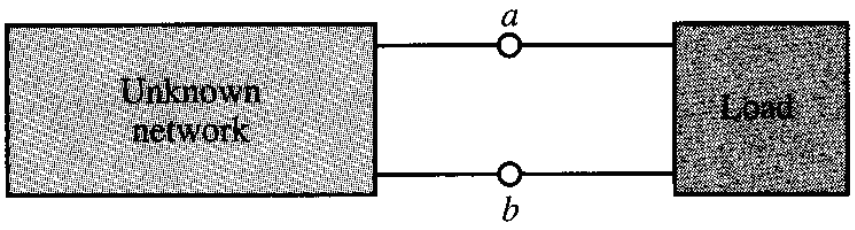
Application 2/2



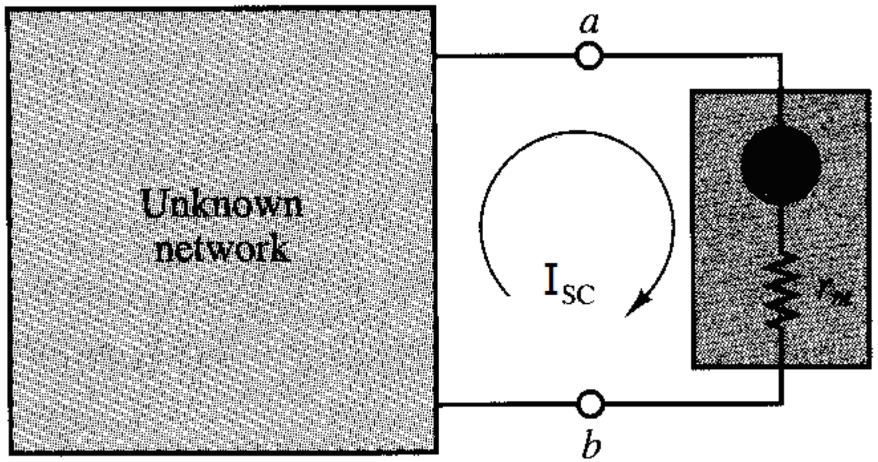
延伸命題：如果使用的ammeter與voltmeter非理想儀器，而是有內部電阻時，結果又如何？

$$R_{TH} = R_N = E_{TH} \div I_N$$

Experimental Determination of Thevenin and Norton Equivalents



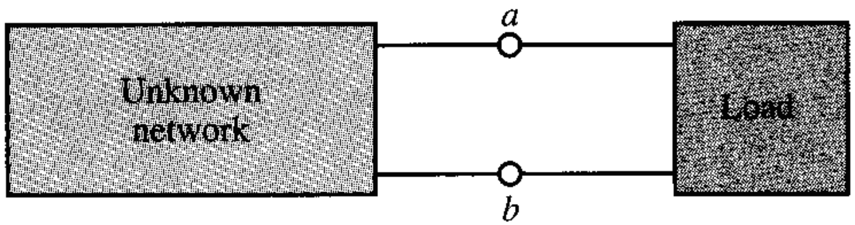
An unknown network connected to a load



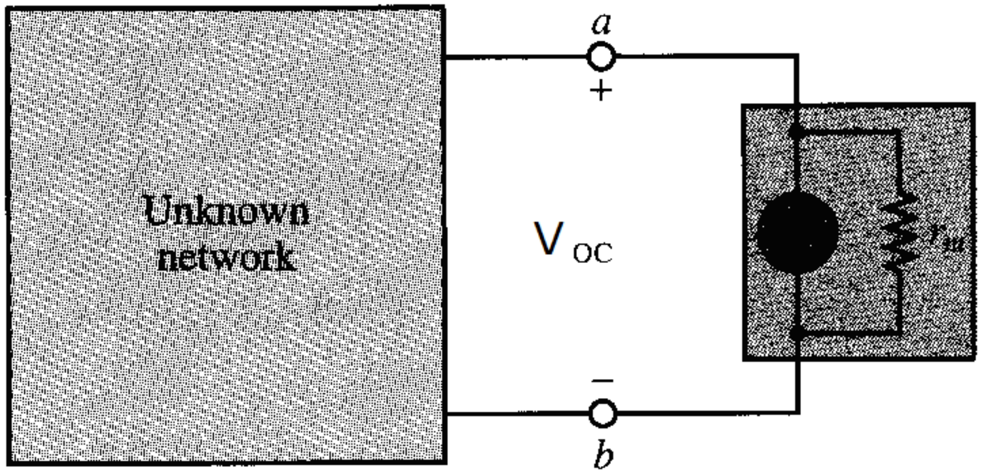
Network connected for measurement of short-circuit current

$$I_N = I_{SC}\left(1 + \frac{r_m}{R_{TH}}\right)$$

Experimental Determination of Thevenin and Norton Equivalents



An unknown network connected to a load



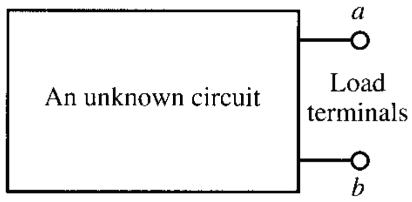
Network connected for measurement of open-circuit voltage

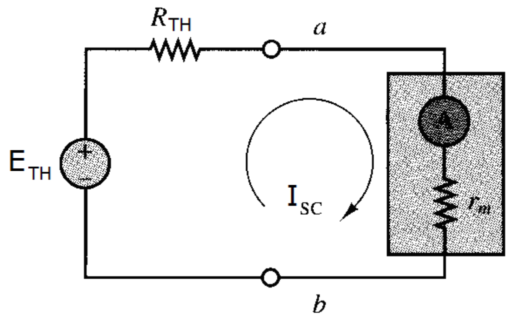
$$E_{TH} = V_{OC}\left(1 + \frac{R_{TH}}{r_m}\right)$$

Example

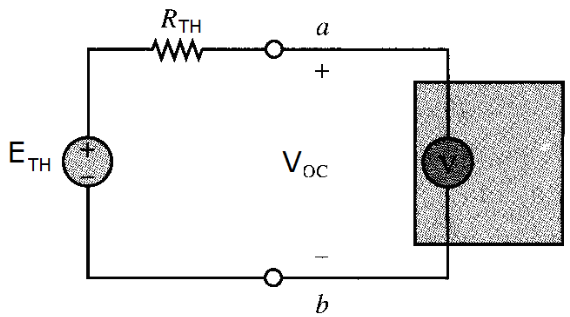
Measured V_{OC}=6.5V
Measured I_{SC}=3.75mA r_m=15Ω

The unknown circuit is replaced by its Thevenin equivalent.





Network connected for measurement of short-circuit current (practical ammeter)



Network connected for measurement of open-circuit voltage (ideal voltmeter)

$$E_{TH} = V_{OC} = 6.5V$$

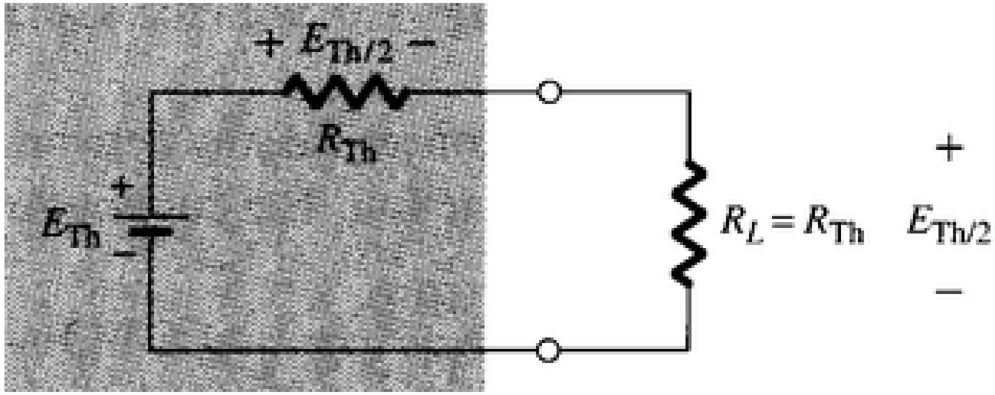
$$I_N = I_{SC}\left(1 + \frac{r_m}{R_{TH}}\right)$$

$$R_{TH=} = \frac{E_{TH}}{I_N} = \frac{V_{OC}}{I_{SC}} - r_m$$

Maximum Power Transfer

- ❑ 確保負載可以從supply收到最多的功率！
- ❑ 當 $R_L = R_{Th}$ 時，負載可以從supply收到最多的功率！

$$R_L = R_{Th}$$



$$I_L = \frac{E_{Th}}{R_L + R_{Th}} = \frac{E_{Th}}{2R_{Th}}$$

$$P_L = I_L^2 R = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 R_{Th}$$

$$P_{L\max} = \frac{E_{Th}^2}{4R_{Th}}$$

當R_L=0.5R_{Th} 或 R_L=1.5 R_{Th}，將發現Power都較低

Exercise 20 Thevenin network

20. a. Find the Thévenin network for the portion of the network in Fig. 3.89 to the left of resistor R_L.

b. Using the Thévenin network, determine I_L.

c. Use series–parallel techniques on the original network to determine I_L, and compare the results with those for part (b).

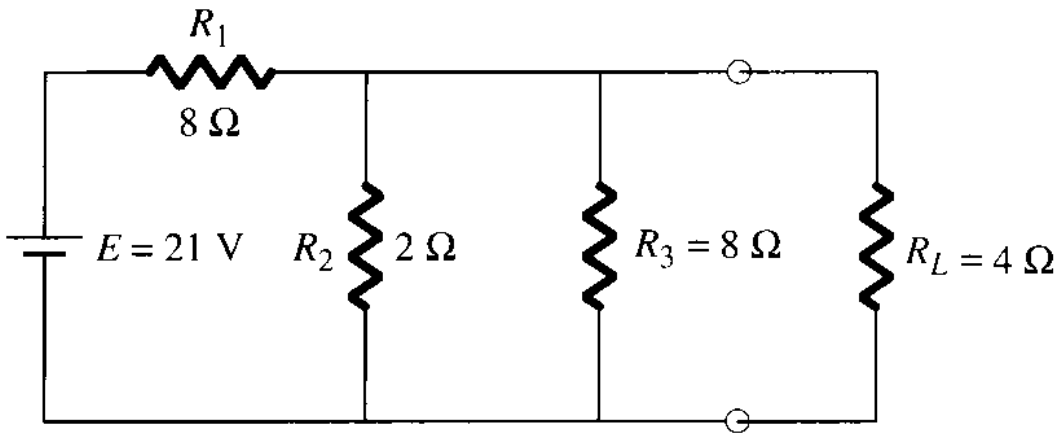
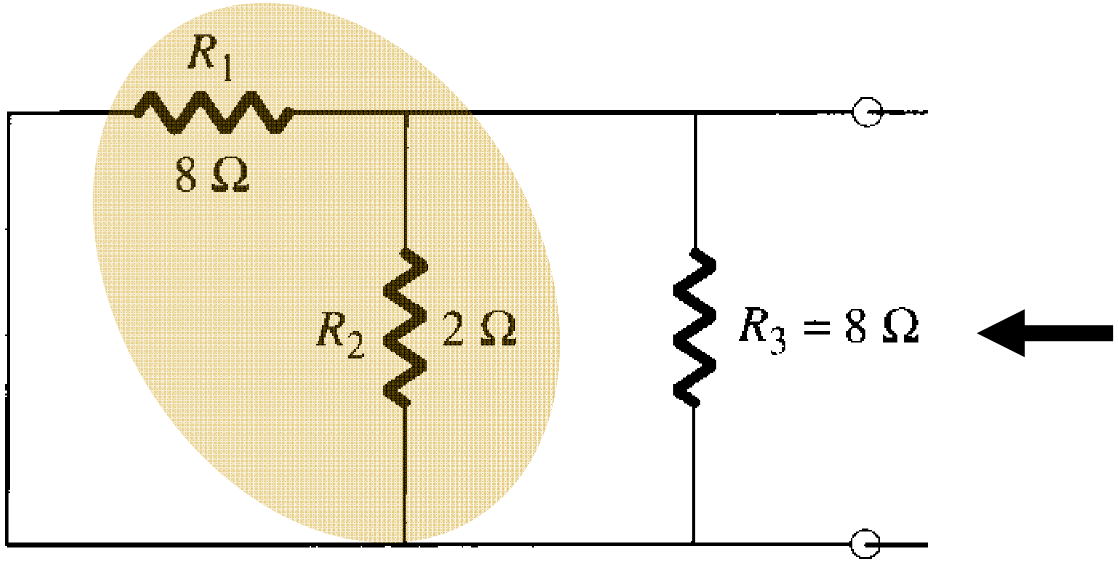


FIG. 3.89

R_{TH}

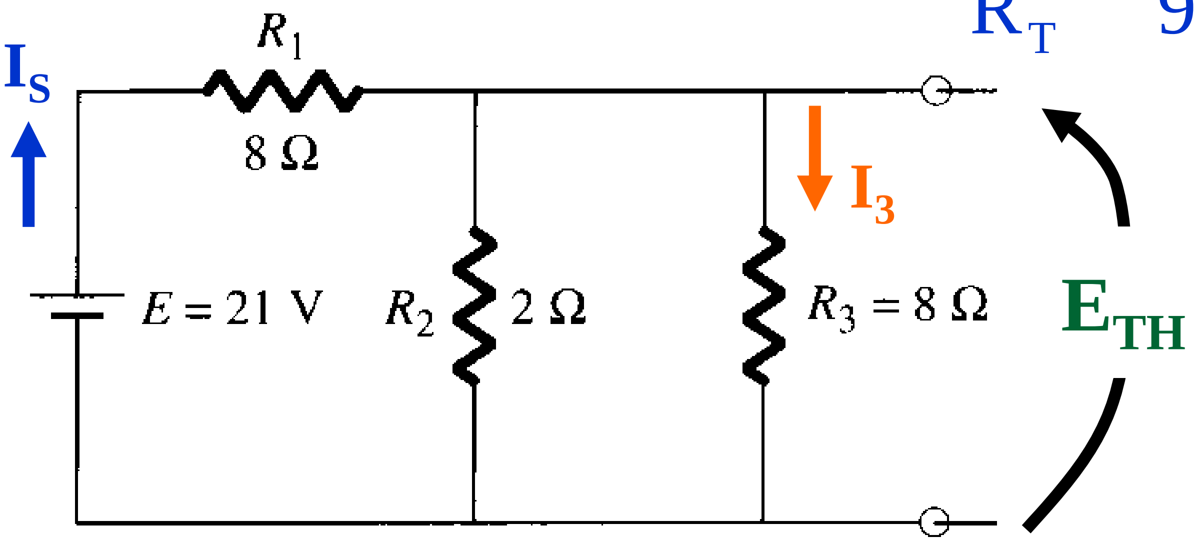


$$R_{TH} = (R_1 // R_2) // R_3 = 1.333\Omega$$

E_{TH}

$$R_T = (R_3 // R_2) + R_1 = 9.6\Omega$$

$$I_s = \frac{E}{R_T} = \frac{21V}{9.6\Omega} = 2.1985A$$

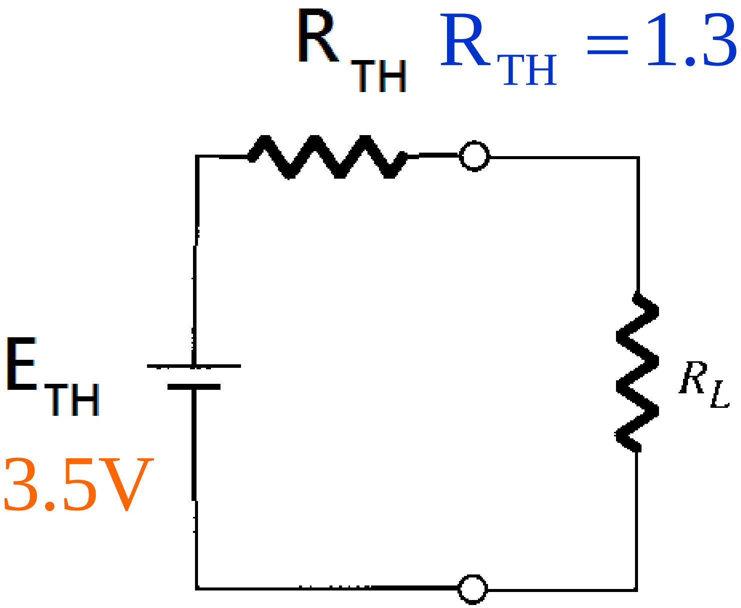


$$I_3 = I_S \times \frac{R_2}{R_2 + R_3} = 0.4375A$$

$$E_{TH} = I_3 \times R_3 = 3.5V$$

Thévenin Network

$$R_{TH} = 1.333\Omega$$



$$E_{TH} = 3.5V$$

$$I_L = \frac{E_{TH}}{R_{TH} + R_L} = \frac{3.5V}{(1.333\Omega + 4\Omega)} = 0.656A$$

$$P = I_L^{\ 2} \times R_L = 1.721W$$

Exercise 21 Thevenin network

21. **a.** Determine the value of R_L in Fig. 3.89 that would result in maximum power to R_L.

b. Calculate the maximum power that could be delivered to R_L if it were changed to that value.

c. Find the power delivered to R_L if $R_L = 4\ \Omega$ and verify that it is less than the maximum value in part (b).

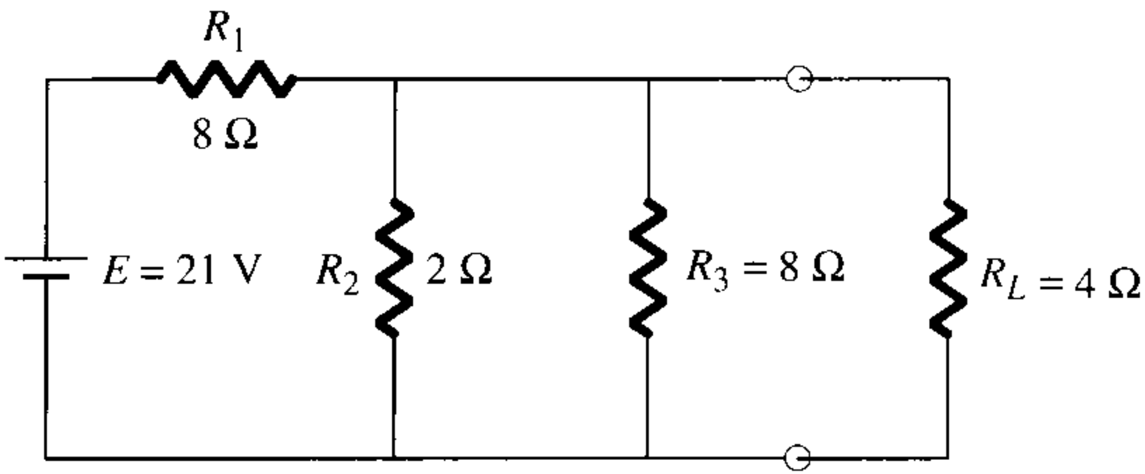


FIG. 3.89

Solution

當$R_L = R_{TH} = 1.333\,\Omega$

$$I_L = \frac{E_{TH}}{R_{TH} + R_L} = \frac{3.5V}{(1.333\Omega + 1.333\Omega)} = 1.3125A$$

$$P = I_L^{\,2} \times R_L = 2.297W = \frac{E_{TH}^{\,2}}{4R_L} = \frac{E_{TH}^{\,2}}{4R_{TH}} = 2.297W$$

$$I_L = \frac{E_{TH}}{R_{TH} + R_L} = \frac{3.5V}{(1.333\Omega + 4\Omega)} = 0.656A$$

$$P = I_L^{\,2} \times R_L = 1.721W$$ （非最大）

Exercise 22 Thevenin network

22. **a.** Find the Thévenin network for the network external to the resistor R_L in Fig. 3.90.

b. Find R_L for maximum power to R_L.

c. Find the maximum power to R_L.

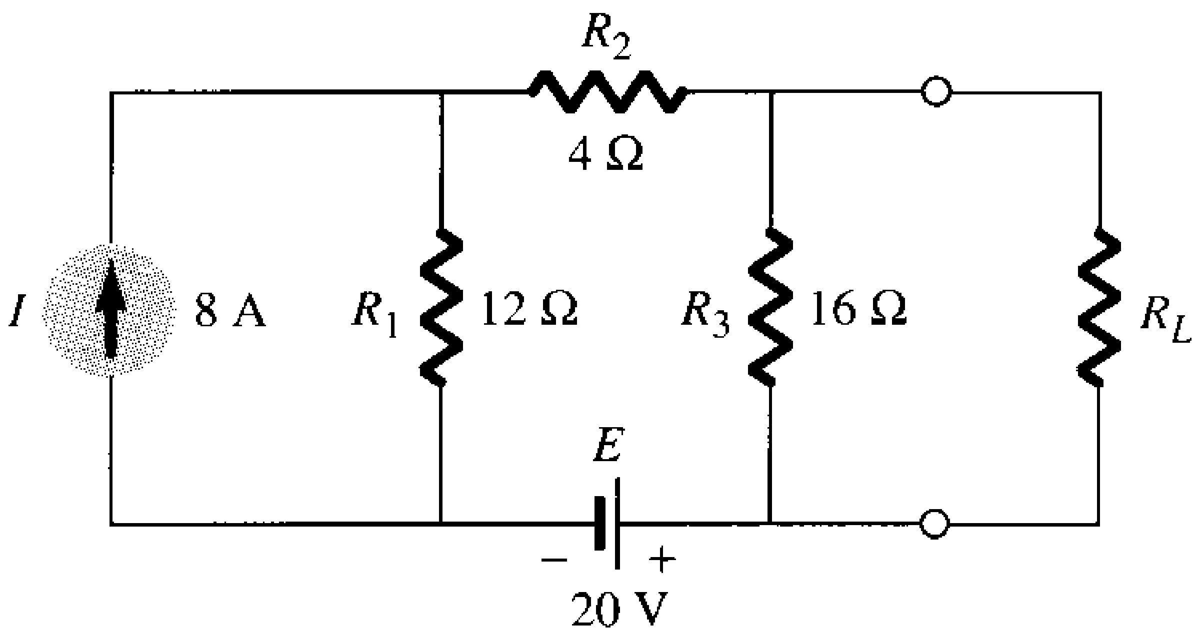
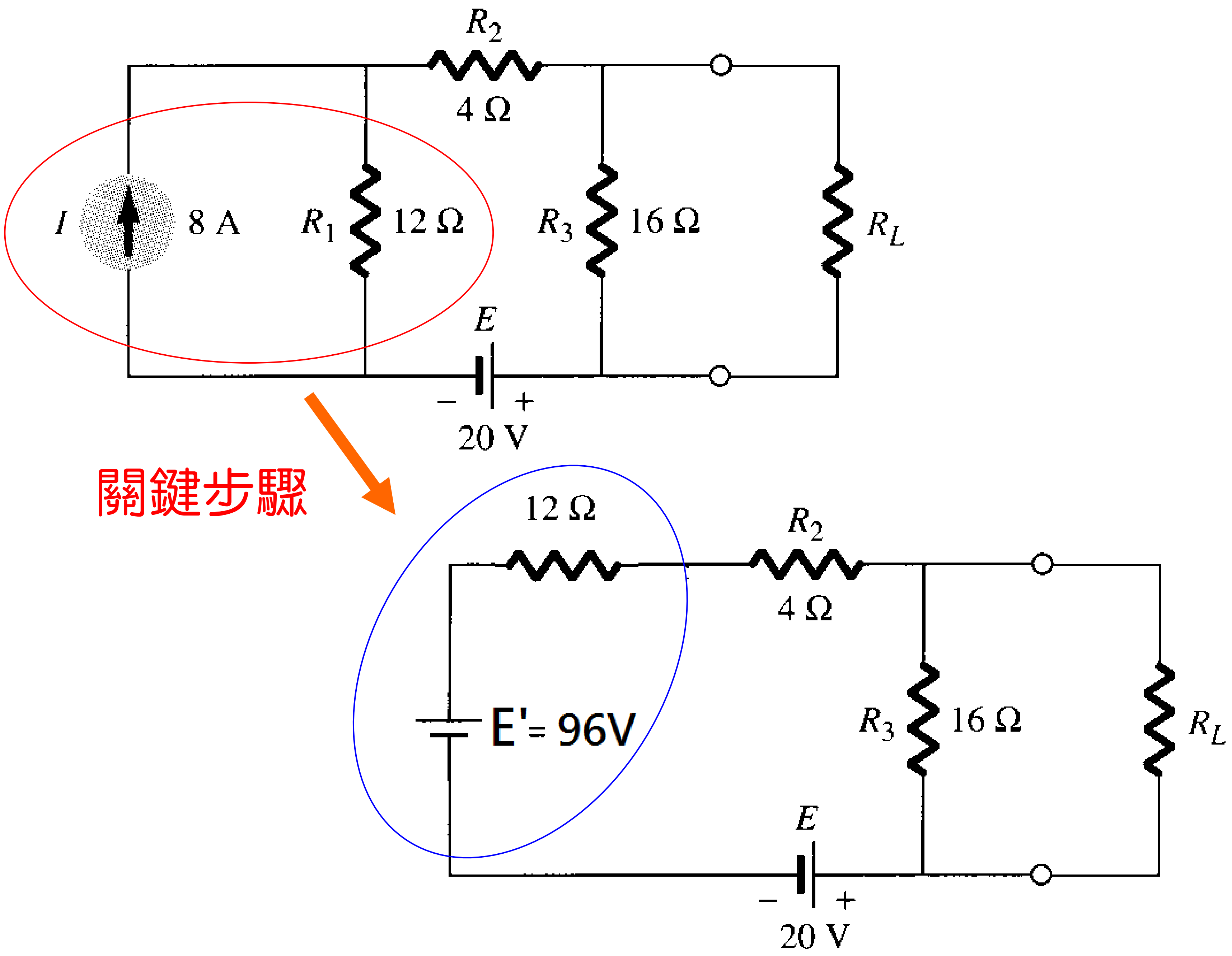
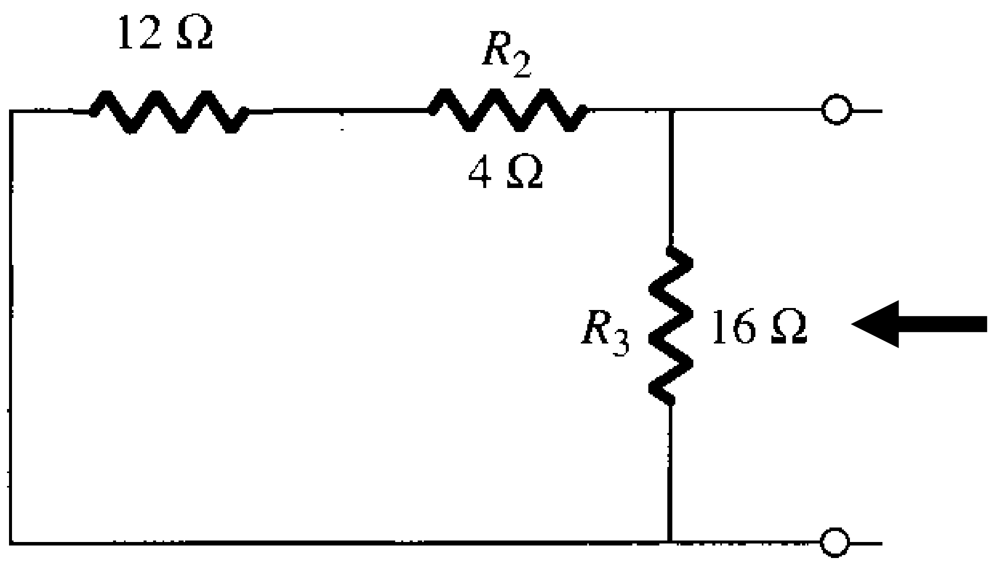


FIG. 3.90



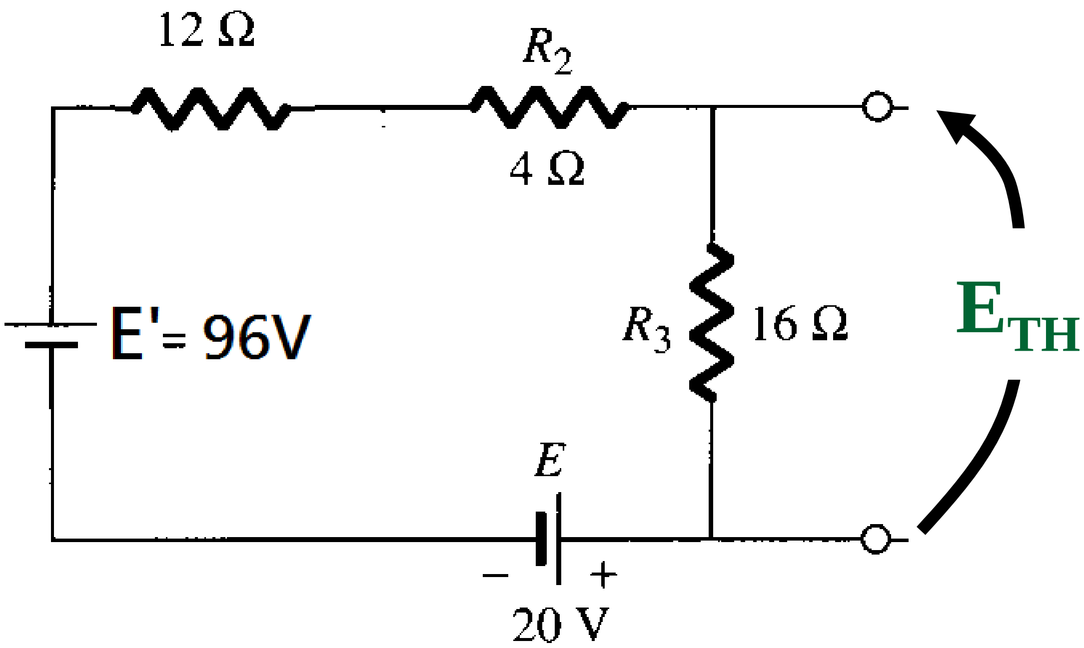
R_2
4 Ω
I
8 A
R_1
12 Ω
R_3
16 Ω
R_L
E
− +
20 V
關鍵步驟
12 Ω
R_2
4 Ω
E'= 96V
R_3
16 Ω
R_L
E
− +
20 V

R_{TH}



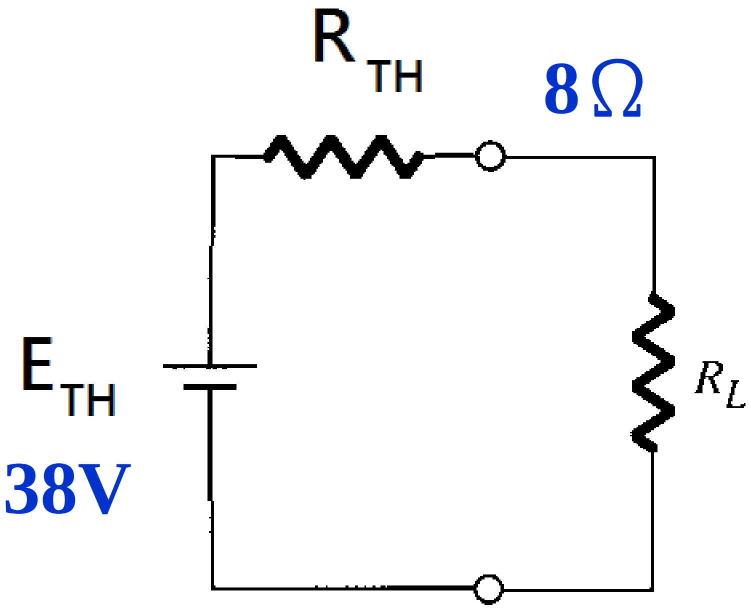
$$R_{TH} = (12\Omega + R_2) // R_3 = 8\Omega$$

E_{TH}



$$E_{TH} = (E' - E) \times \frac{R_3}{12\Omega + R_2 + R_3} = 38V$$

Thévenin Network



當$R_L = R_{TH} = 8\Omega$

$$P_{max} = \frac{E_{TH}^2}{4R_L} = \frac{E_{TH}^2}{4R_{TH}} = 45.125W$$