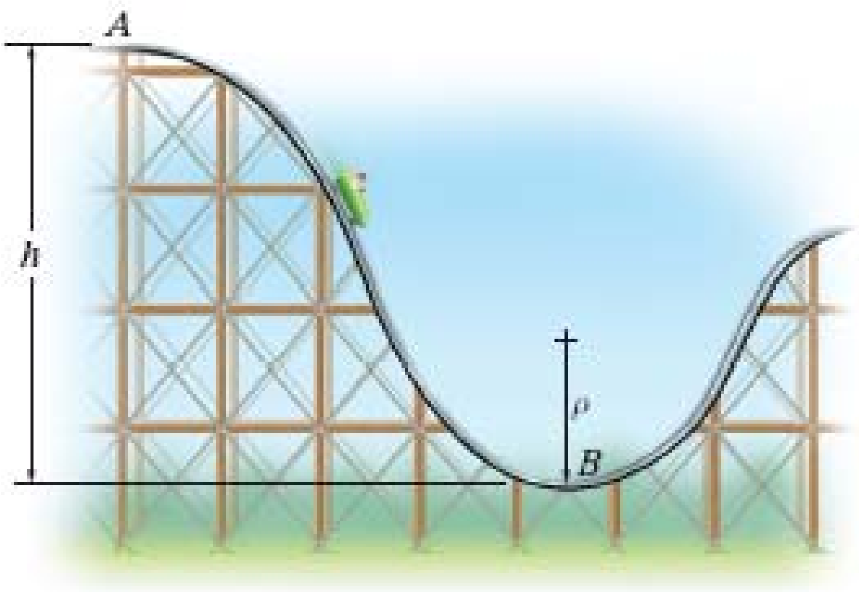
Chap. 14
Kinetics of particles: work and energy

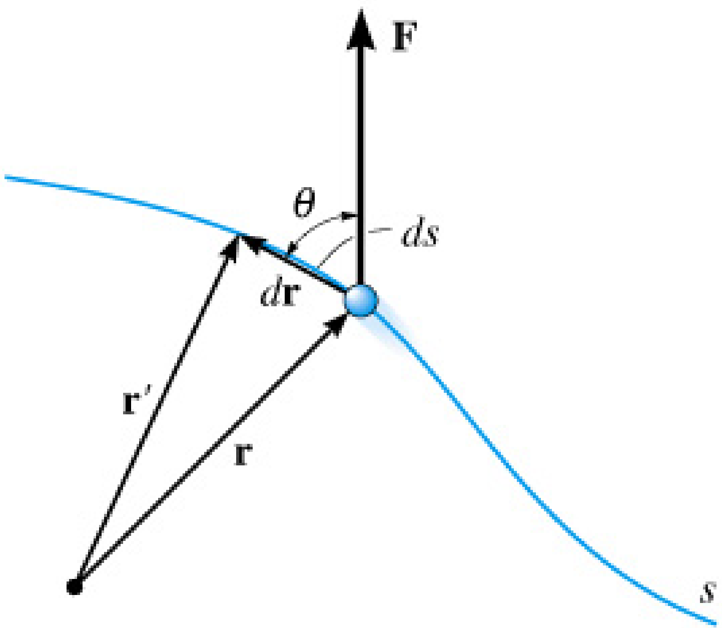
APPLICATIONS



A roller coaster makes use of gravitational forces to assist the cars in reaching high speeds in the “valleys” of the track.

How can we design the track (e.g., the height, h, and the radius of curvature, ρ) to control the forces experienced by the passengers?

14.1 Work of a Force



質點P的位移：

*d***r** = **r'** - **r**

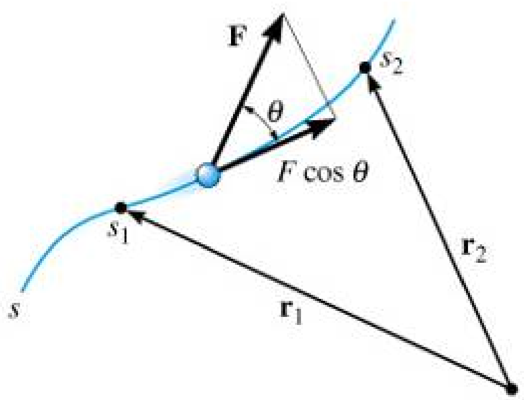
外力F所做的功：

$$dU = \underline{F} \cdot d\,\underline{r}$$

$$= Fds\cos\theta$$

單位： J (N-m) or (ft-lb)

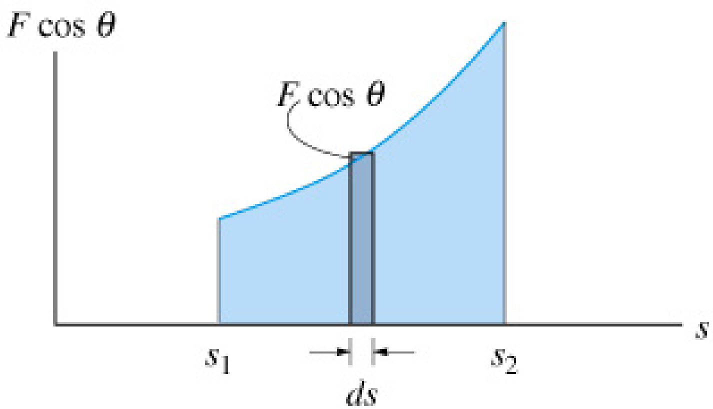
Work of a Variable Force



(a)

$$U_{1\to 2} = \int_{r_1}^{r_2} \underline{F} \cdot d\,\underline{r}$$

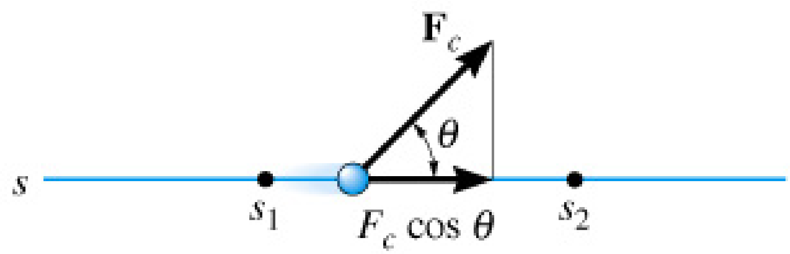
$$= \int_{S_1}^{S_2} F\cos\theta \, ds$$



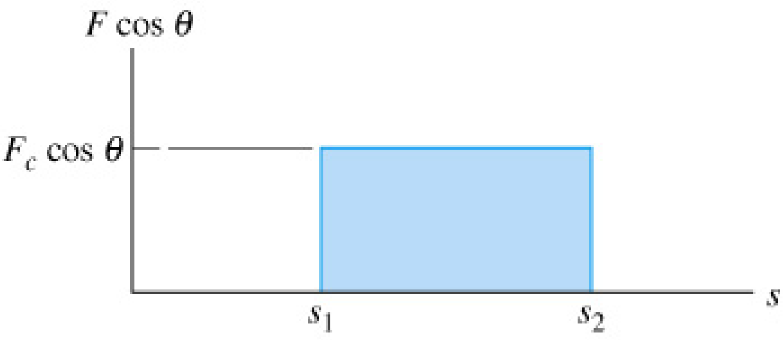
(b)

運動位移由S_1到S_2的曲線下之面積

Work of a Constant Force (Fc)



(a)



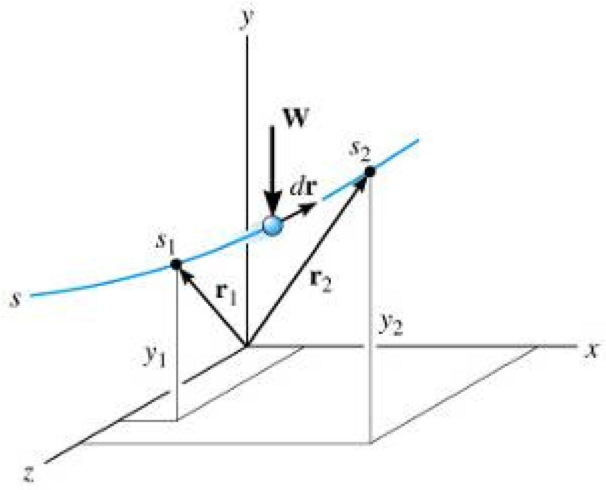
(b)

- 運動軌跡為直線 $U_{1\rightarrow 2} = F_C \cos\theta \int_{S_1}^{S_2} ds$

$$= F_C \cos\theta (S_2 - S_1)$$

(矩形面積)

Work of Weight



重力：$\underline{W} = -W\underline{j}$

位移：$d\underline{r} = dx\underline{i} + dy\underline{j} + dz\underline{k}$

$$U_{1\to 2} = \int \underline{F} \cdot d\underline{r} = \int_{r_1}^{r_2} (-W\underline{j}) \cdot (dx\underline{i} + dy\underline{j} + dz\underline{k})$$

$$= \int_{y_1}^{y_2} -W dy$$

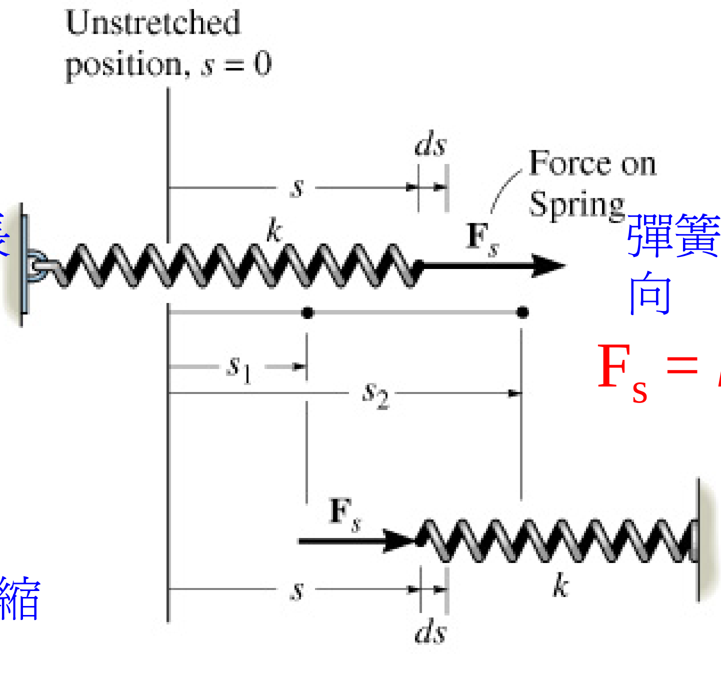
$$= -W(y_2 - y_1)$$

or

$U_{1\to 2} = -W\Delta y$ (往下做運動時做正功)

Work of a Spring Force 彈簧力Fs

伸長



(a)

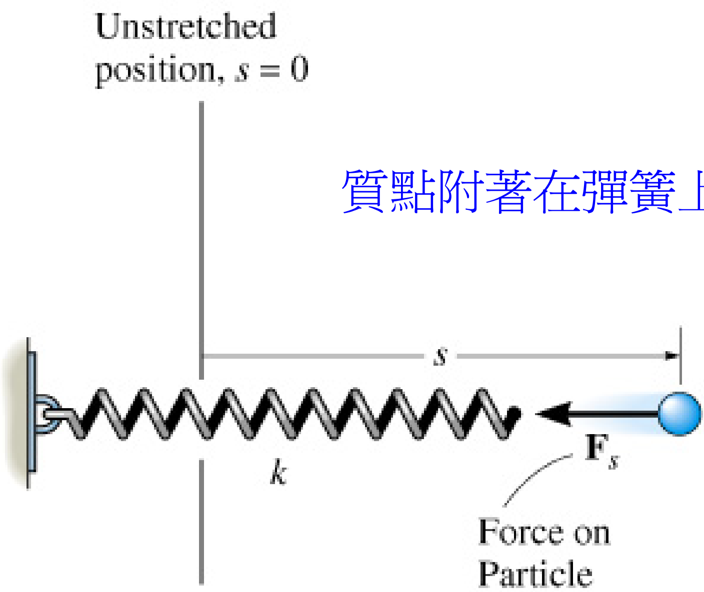
壓縮

彈簧力與位移同向

$F_s = ks$

$$U_{1\to 2} = \int_{S_1}^{S} F_S \, ds = \int_{S_1}^{S_2} ks \, ds$$

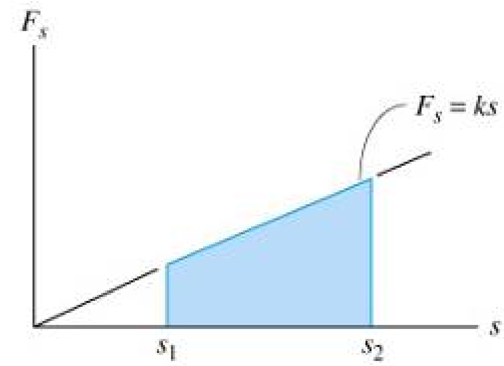
$$= \frac{1}{2} k s_2^{\,2} - \frac{1}{2} k s_1^{\,2}$$



質點附著在彈簧上

$F_s = -ks$

(c) 質點受力與位移反向



(b)

$$U_{1\to 2} = -\left(\frac{1}{2}ks_2^{\,2} - \frac{1}{2}ks_1^{\,2}\right)$$

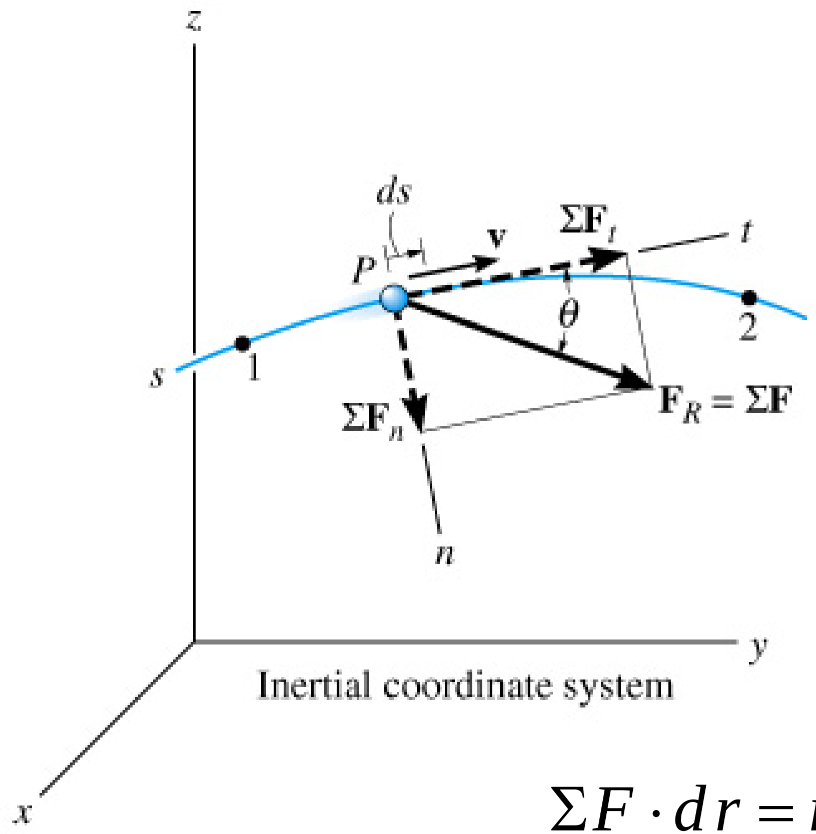
14.2
Principle of Work and Energy

$$a = \frac{dv}{dt} \cdot \frac{ds}{ds}$$

$$= \frac{dv}{ds} \cdot \frac{ds}{dt}$$

$$= v\frac{dv}{ds}$$

$$ads = vdv$$



$$\Sigma \underline{F} \cdot d\,\underline{r} = m\underline{a} \cdot d\,\underline{r}$$

僅有切線方向的ΣF_t有做功

$$\Sigma \underline{F} \cdot d\underline{r} = \Sigma \underline{F} ds\cos\theta = \Sigma F_t ds = m\underline{a}_t ds$$

$$\Rightarrow \Sigma \int_{r_1}^{r_2} \underline{F} \cdot d\underline{r} = \int_{v_1}^{v_2} mvdv \qquad (\because a_t ds = vdv)$$

$$= \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2$$

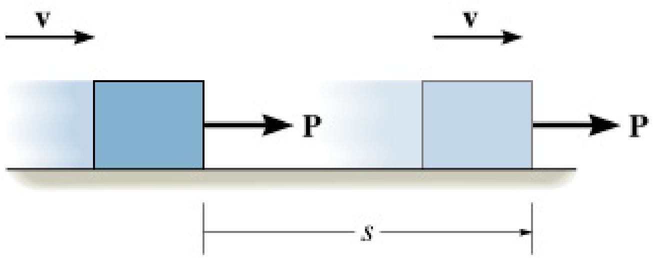
$$\Rightarrow \Sigma U_{1\to 2} = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2 \qquad \leftarrow \text{動能定律}$$

設動能 $\mathrm{T} = \frac{1}{2}mv^2$

$$\Rightarrow T_1 + \Sigma U_{1\to 2} = T_2$$

(僅能求切線方向的力)

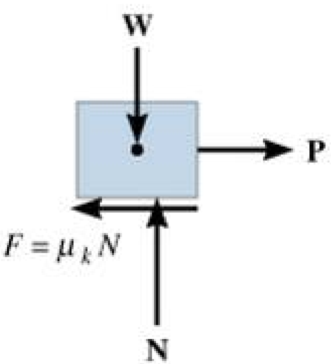
Work of Friction Caused by Sliding



(a)

$$\frac{1}{2}mv^2 + Ps - \mu_k Ns = \frac{1}{2}mv^2$$

實際上 $\mu_k N$ 的位移S'應小於S



(b)


(c)

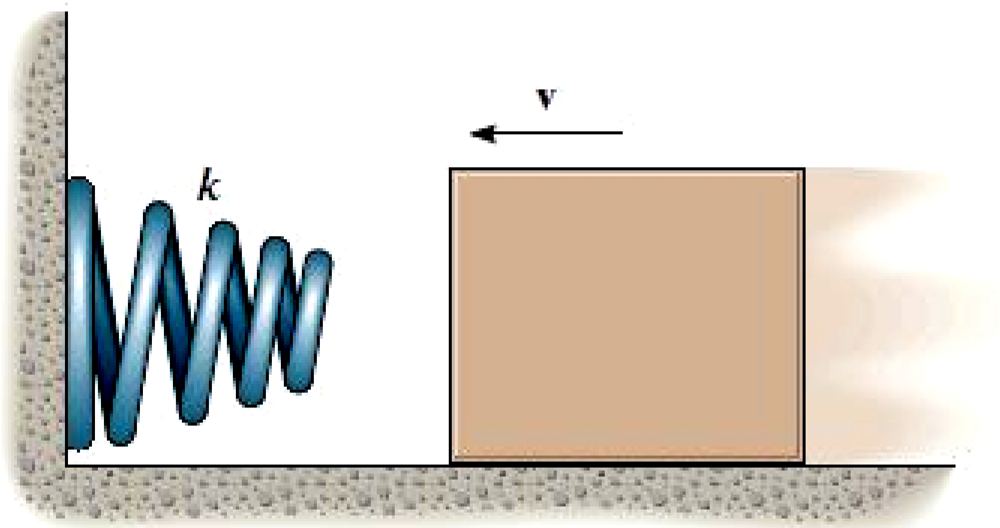
其中的差值

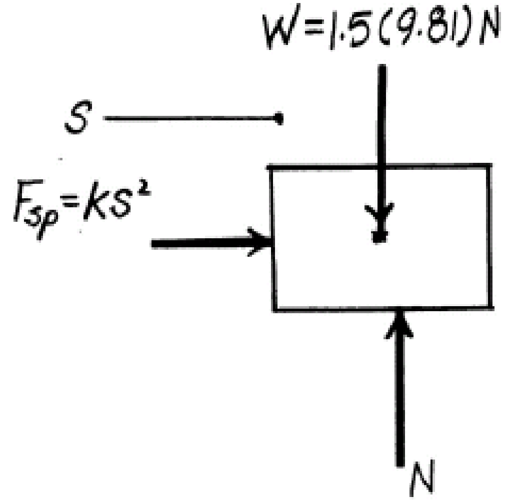
$\mu_k N(S - S')$ 爲內能

使物體的溫度上升

p. 184, 14-5

The 1.5-kg block slides along a smooth plane and strikes a *nonlinear spring* with a speed of v = 4 m/s. The spring is termed “nonlinear” because it has a resistance of $F_S = ks^2$, where k = 900 N/m^2. Determine the speed of the block after it has compressed the spring s = 0.2 m.





Principle of Work and Energy: The spring force F_{sp} which acts in the opposite direction to that of displacement does *negative* work. The normal reaction N and the weight of the block do not displace hence do no work. Applying Eq. 14–7, we have

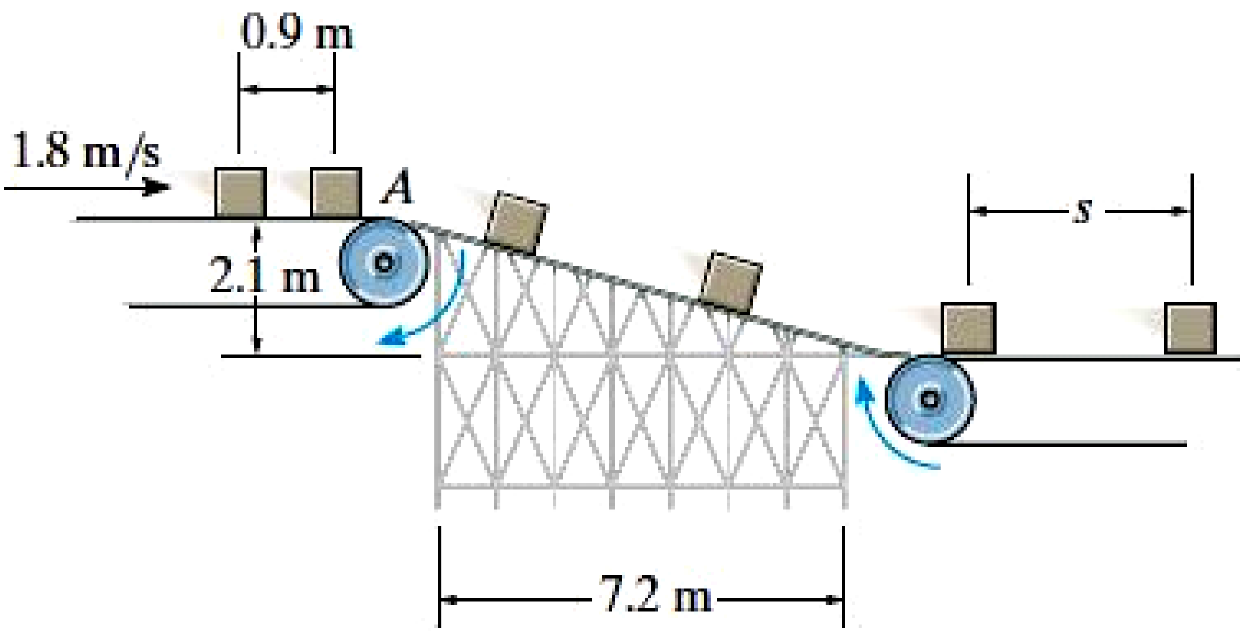
$$T_1 + \sum U_{1-2} = T_2$$

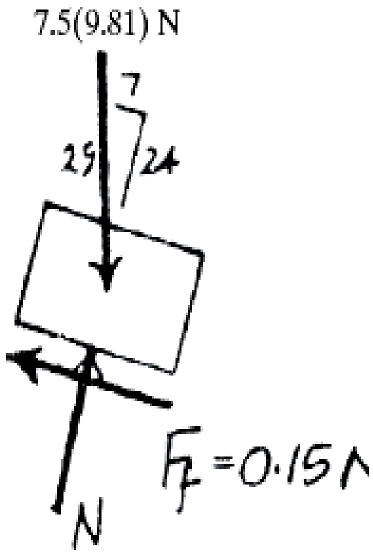
$$\frac{1}{2}(1.5)\left(4^2\right) + \left[-\int_0^{0.2\text{ m}} 900s^2\,ds\right] = \frac{1}{2}(1.5)\,v^2$$

$$v = 3.58\text{ m/s} \qquad \textbf{Ans.}$$

p. 187, 14-20

Packages having a mass of 7.5 kg are transferred horizontally from one conveyor to the next using a ramp for which $\mu_k = 0.15$. The top conveyor is moving at 1.8 m/s and the packages are spaced 0.9 m apart. Determine the required speed of the bottom conveyor so no sliding occurs when the packages come horizontally in contact with it. What is the spacing *s* between the packages on the bottom conveyor?





Equations of Motion:

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 7.5(9.81)\left(\frac{24}{25}\right) = 7.5(0) \qquad N = 70.63\text{ N}$$

Principle of Work and Energy: Only force components parallel to the inclined plane which are in the direction of displacement [7.5(9.81)(7/25) N and $F_f = \mu_k N$ = 0.15(70.63) = 10.59 N] do work, whereas the force components perpendicular to the inclined plane [7.5(9.81)(24/25) N and normal reaction N] do no work since no displacement occurs in this direction. Here, the 7.5(9.81)(7/25) N force does *positive* work and F_f = 10.59 N does *negative* work. Slipping at the contact surface between the package and the belt will not occur if the speed of belt is the same as the speed of the package at B. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(7.5)(1.8^2) + 7.5(9.81)\left(\frac{7}{25}\right)\left(\sqrt{2.1^2 + 7.2^2}\right) - 10.59\left(\sqrt{2.1^2 + 7.2^2}\right) = \frac{1}{2}(7.5)v^2$$

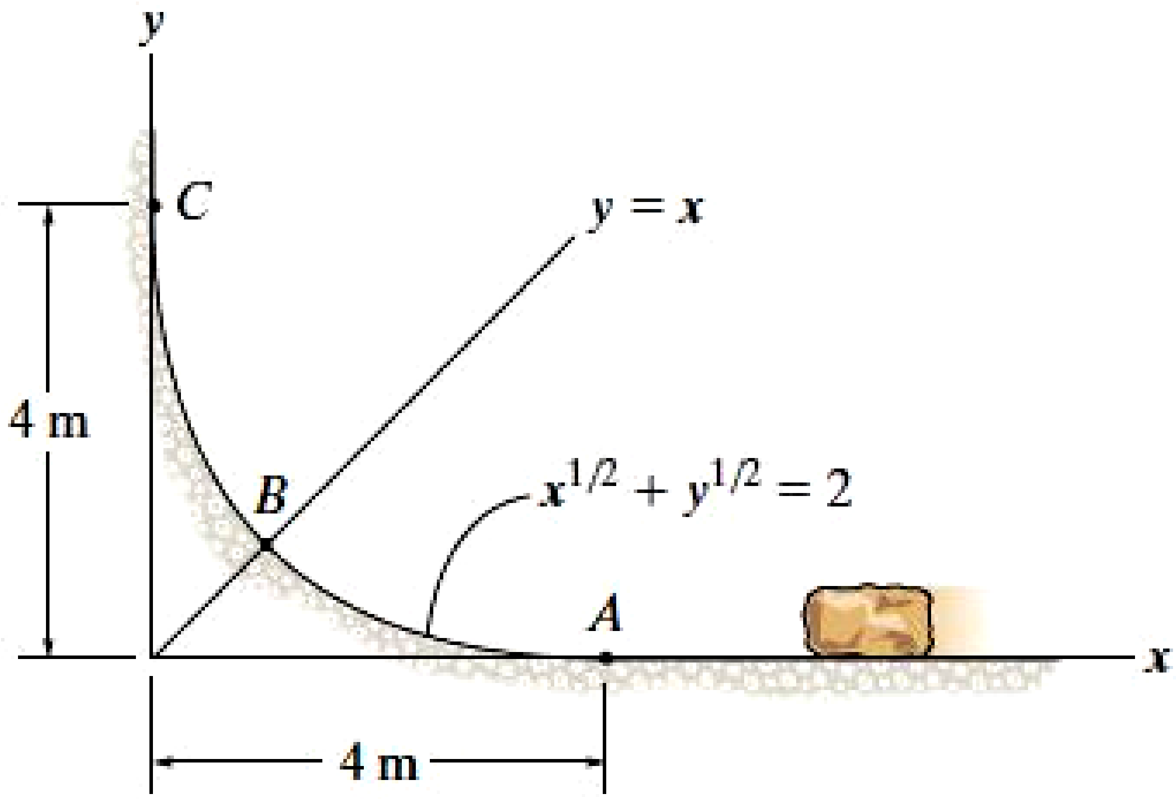
$$v = 4.823\text{ m/s}$$ **Ans.**

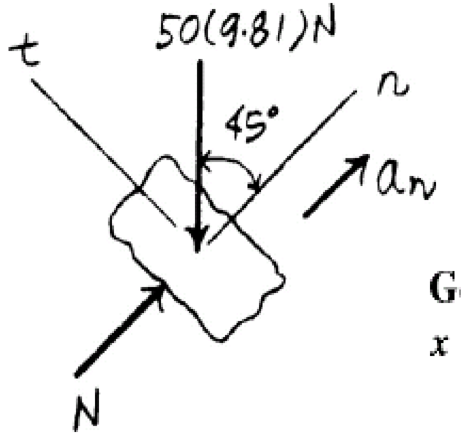
The time between two succesive packages to reach point B is $t = \dfrac{0.9}{1.8} = 0.5$ s. Hence, the distance between two succesive packages on the lower belt is

$$s = vt = 4.823(0.5) = 2.412\text{ m}$$ **Ans.**

p. 191, 14-36

The 50-kg stone has a speed of v_A = *8 m/s* when it reaches point *A*. Determine the normal force it exerts on the incline when it reaches point *B*. Neglect friction and the stone's size.





Geometry: Here, $x^{1/2} + y^{1/2} = 2$. At point B, $y = x$, hence $2x^{1/2} = 2$ and $x = y = 1$ m.

$$y^{-1/2}\frac{dy}{dx} = -x^{-1/2} \qquad \frac{dy}{dx} = \left.\frac{x^{-1/2}}{y^{-1/2}}\right|_{x=1\text{ m},\, y=1\text{ m}} = -1$$

$$y^{-1/2}\frac{d^2y}{dx^2} + \left(-\frac{1}{2}\right)y^{-3/2}\left(\frac{dy}{dx}\right)^2 = -\left(-\frac{1}{2}x^{-3/2}\right)$$

$$\frac{d^2y}{dx^2} = y^{1/2}\left.\left[\frac{1}{2y^{3/2}}\left(\frac{dy}{dx}\right)^2 + \frac{1}{2x^{3/2}}\right]\right|_{x=1\text{ m},\, y=1\text{ m}} = 1$$

The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=1\text{ m},\, y=1\text{ m}} = -1 \qquad \theta = -45.0^\circ$$

and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (-1)^2\right]^{3/2}}{|1|}\right| = 2.828\text{ m}$$

Principle of Work and Energy: The weight of the block which acts in the opposite direction to that of the vertical displacement does *negative* work when the block displaces 1 m vertically. Applying Eq. 14–7, we have

$$T_A + \sum U_{A-B} = T_B$$

$$\frac{1}{2}(50)\left(8^2\right) - 50(9.81)(1) = \frac{1}{2}(50)\,v_B^2$$

$$v_B^2 = 44.38\ \text{m}^2/\text{s}^2$$

Equations of Motion: Applying Eq. 13–8 with $\theta = 45.0°$, $v_B^2 = 44.38\ \text{m}^2/\text{s}^2$ and $\rho = 2.828$ m, we have

$$+\nearrow\Sigma F_n = ma_n; \qquad N - 50(9.81)\cos 45° = 50\left(\frac{44.38}{2.828}\right)$$

$$N = 1131.37\ \text{N} = 1.13\ \text{kN} \qquad \textbf{Ans.}$$

14.4 Power and Efficiency

Def. $P = \frac{dU}{dt}$ (功對時間的變化率)

$= \frac{\underline{F} \cdot d\underline{r}}{dt}$

$= \underline{F} \cdot \underline{v}$

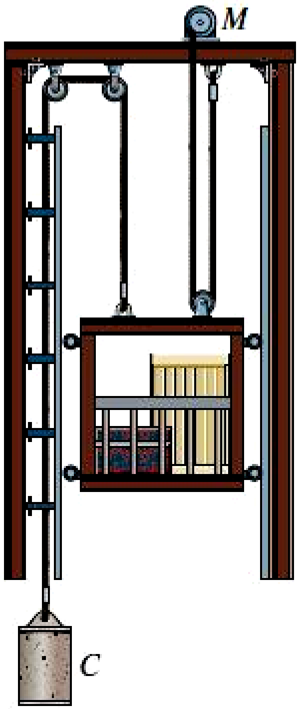
單位：W (watt) = J/s = N-m/s

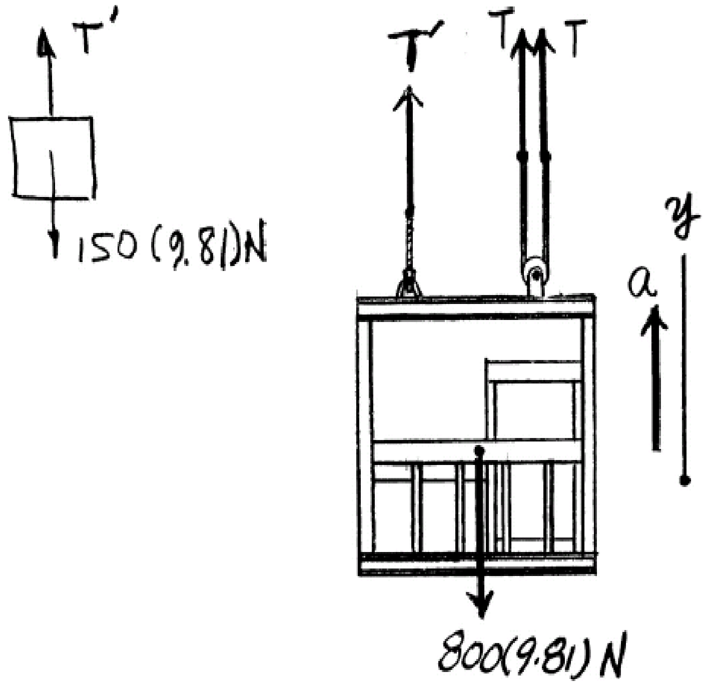
1 hp = 550 ft-lb/s= 746 W

效率 $\varepsilon = \frac{\text{輸出的功率}}{\text{輸入的功率}} = \frac{\text{輸出的能量}}{\text{輸入的能量}} \quad (<1)$

p. 197, 14-52

The material hoist and the load have a total mass of 800 kg and the counterweight *C* has a mass of 150 kg. If the upward speed of the hoist increases uniformly from 0.5 m/s to 1.5 m/s in 1.5 s. Determine the average power generated by the motor *M* during this time. The motor operates with an efficiency of . $\varepsilon = 0.8$





Kinematics: The acceleration of the hoist can be determined from

$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$1.5 = 0.5 + a(1.5)$$

$$a = 0.6667 \text{ m/s}^2$$

Equations of Motion: Using the result of **a** and referring to the free-body diagram of the hoist and block shown in Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(0.6667)$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 150\left(9.81\right) - T' = 150(0.6667)$$

Solving,

$$T' = 1371.5 \text{ N}$$

$$T = 3504.92 \text{ N}$$

Power:

$$(P_{\text{out}})_{\text{avg}} = 2\mathbf{T} \cdot \mathbf{v}_{\text{avg}} = 2(3504.92)\left(\frac{1.5 + 0.5}{2}\right) = 7009.8 \text{ W}$$

Thus,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{7009.8}{0.8} = 8762.3 \quad \text{W} = 8.76 \text{ kW}$$ **Ans.**

14.5 Conservative Forces and Potential Energy

Conservative Forces (保守力)

Def. 施於一質點的外力所做的功與路徑無關，只是和位置有關，則此外力稱為一**保守力**。

Ex. 重力、彈簧力

$$U_w = -w\Delta y$$

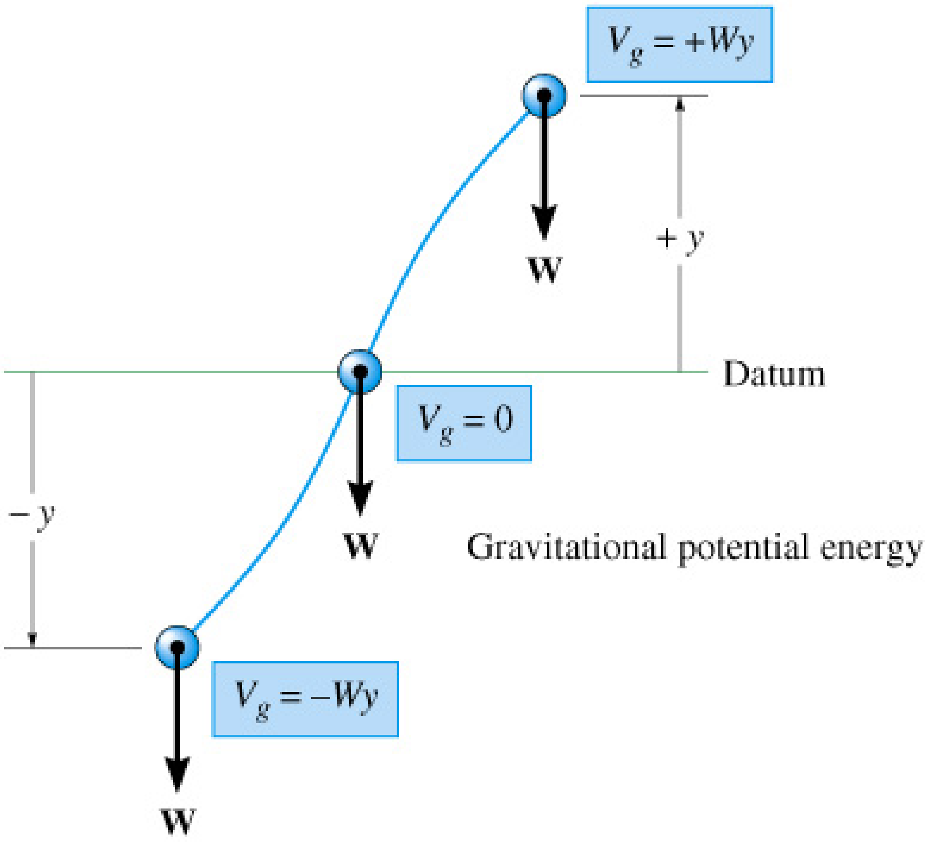
$$U_s = -(\frac{1}{2}k{s_2}^2 - \frac{1}{2}k{s_1}^2)$$

摩擦力：non-conservative force 非保守力

Potential Energy (位能)

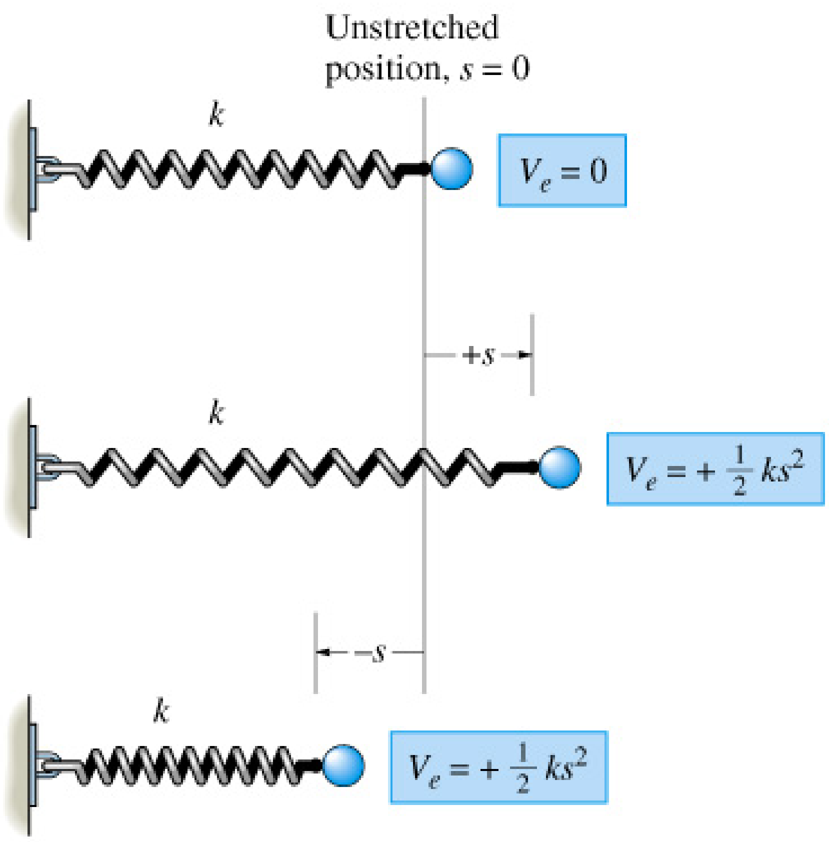
Def. 一保守力由一已知位置至基準線所做的功。

Gravitational Potential Energy (重力位能)



$$V_g = Wy$$

Elastic Potential Energy (彈力位能)

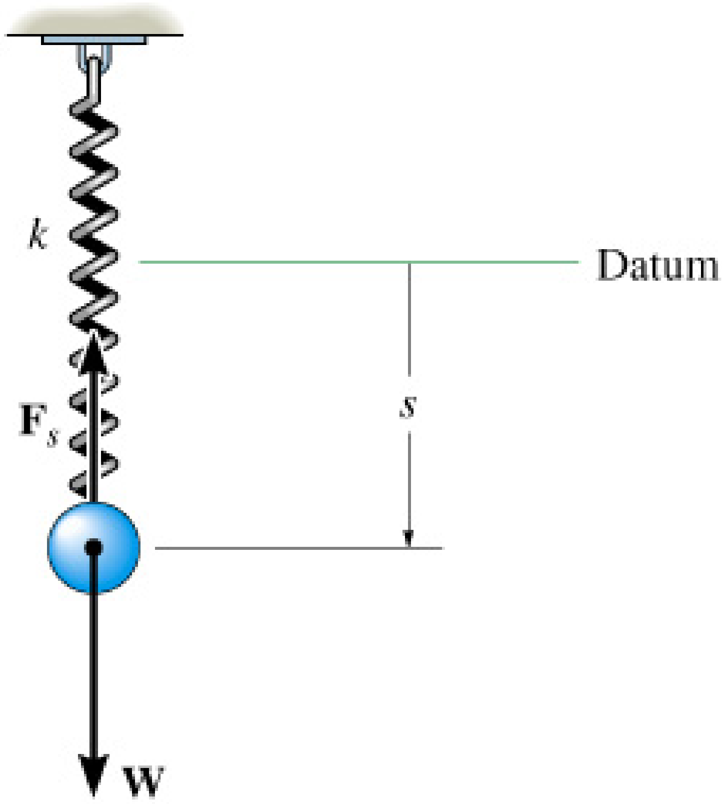


Elastic potential energy

$$V_e = \frac{1}{2}ks^2 \qquad (>0)$$

彈簧力作用於質點的功
(使彈簧回復原狀)

Potential Function (位能函數)



$$V = V_g + V_e$$

$$= -Ws + \frac{1}{2}ks^2$$

Potential Function (位能函數)

$$U_{1\to 2} = V_1 - V_2 = (-Ws_1 + \frac{1}{2}ks_1{}^2) - (-Ws_2 + \frac{1}{2}ks_2{}^2)$$

$$= W(s_2 - s_1) - (\frac{1}{2}ks_2{}^2 - \frac{1}{2}ks_1{}^2)$$

$$dU = V(x, y, z) - V(x + dx, y + dy, z + dz)$$

$$= -dV(x, y, z)$$

Potential Function (位能函數)

又 $$dU = \underline{F} \cdot d\underline{r} = (F_x, F_y, F_z) \cdot (dx, dy, dz)$$

$$= F_x dx + F_y dy + F_z dz$$

$$\Rightarrow F_x dx + F_y dy + F_z dz = -dV(x, y, z)$$

$$= -(\frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz)$$

Potential Function (位能函數)

$$\Rightarrow F_x = -\frac{\partial V}{\partial x} \;,\; F_y = -\frac{\partial V}{\partial y} \;,\; F_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \underline{F} = -\frac{\partial V}{\partial x}\underline{i} - \frac{\partial V}{\partial y}\underline{j} - \frac{\partial V}{\partial z}\underline{k}$$

$$= -(\frac{\partial}{\partial x}\underline{i} + \frac{\partial}{\partial y}\underline{j} + \frac{\partial}{\partial z}\underline{k})V$$

$$\nabla(del) = (\frac{\partial}{\partial x})\underline{i} + (\frac{\partial}{\partial y})\underline{j} + (\frac{\partial}{\partial z})\underline{k}$$

$$\Rightarrow \underline{F} = -\nabla V$$

(保守力需符合此條件)

$$\mathrm{V}_{\mathrm{w}} = Wy$$

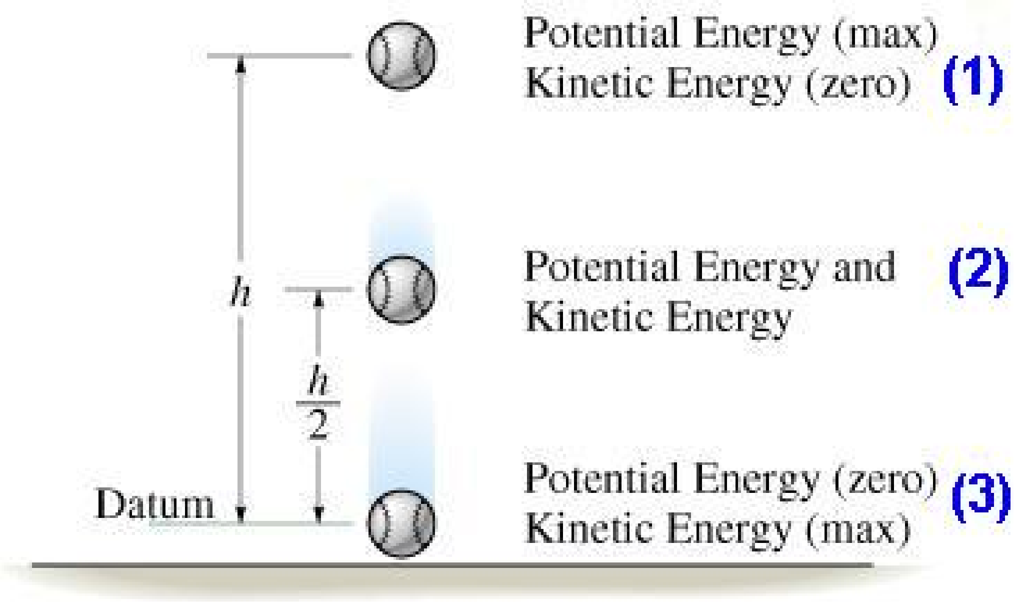
$$\underline{F} = \underline{W} = W\,\underline{j}$$

$$\underline{F} = -(\frac{\partial}{\partial y}\underline{j})Wy$$

$$= -W\,\underline{j}$$

14.6 Conservation of Energy
(能量守恆)

14.6 Conservation of Energy



$$T_1 + V_1 + (\Sigma U_{1\to 2})_{noncons} = T_2 + V_2$$

若$(\Sigma U_{1\to 2})_{noncons} = 0$

則$T_1 + V_1 = T_2 + V_2$

14.6 Conservation of Energy

由 p.174, eq 14-7

$$T_1 + \Sigma U_{1\to 2} = T_2$$
$$\Rightarrow T_1 + (\Sigma U_{1\to 2})_{cons} + (\Sigma U_{1\to 2})_{noncons} = T_2$$
$$\Rightarrow T_1 + (V_1 - V_2) + (\Sigma U_{1\to 2})_{noncons} = T_2$$
$$\Rightarrow T_1 + V_1 + (\Sigma U_{1\to 2})_{noncons} = T_2 + V_2$$

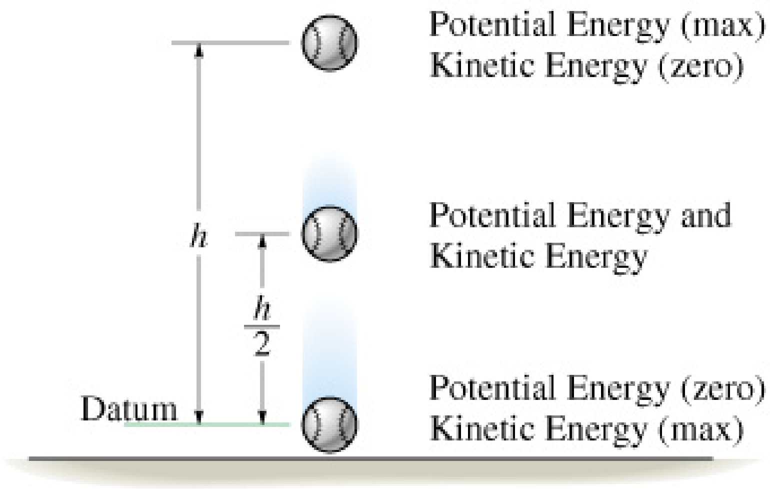
$ex.\ fig.14-20$

(1) $E_1 = V_1 + T_1 = Wh + 0 = Wh$

(2) $E_2 = T_2 + V_2 = \frac{1}{2}\frac{W}{g}(\sqrt{gh})^2 + W\frac{h}{2} = Wh$

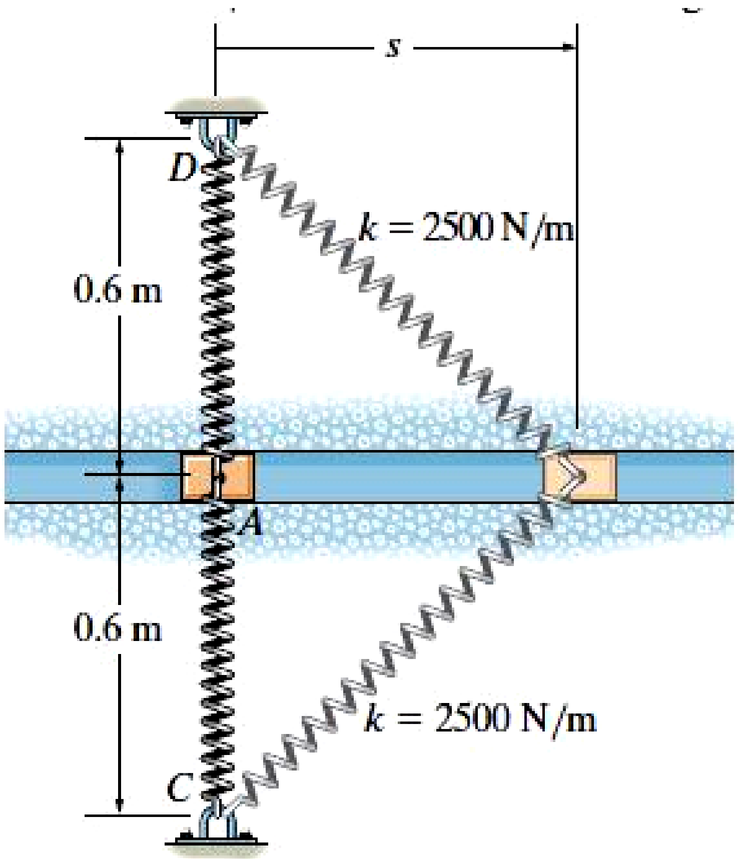
(3) $E_3 = V_3 + T_3 = 0 + \frac{1}{2}\frac{W}{g}(\sqrt{2gh})^2 = Wh$

$\Rightarrow E_1 = E_2 = E_3$



p. 211, 14-80

The 1.5-kg block *A* slides in the smooth horizontal slot. When *s* = 0 the block is given an initial velocity of 18 m/s to the right. Determine the maximum horizontal displacement *s* of the block. Each of the two springs has a stiffness of *k* = 2500 N/m and an unstretched length of 0.15 m.



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(1)(18)^2 + 2\left[\frac{1}{2}(2500)(0.6 - 0.15)^2\right] = 0 + 2\left[\frac{1}{2}(2500)\left(\sqrt{(0.6)^2 + s^2} - 0.15\right)^2\right]$$

Set $d = \sqrt{(0.6)^2 + s^2}$ then

$$d^2 - 0.3d - 0.2448 = 0$$

Solving for the positive root,

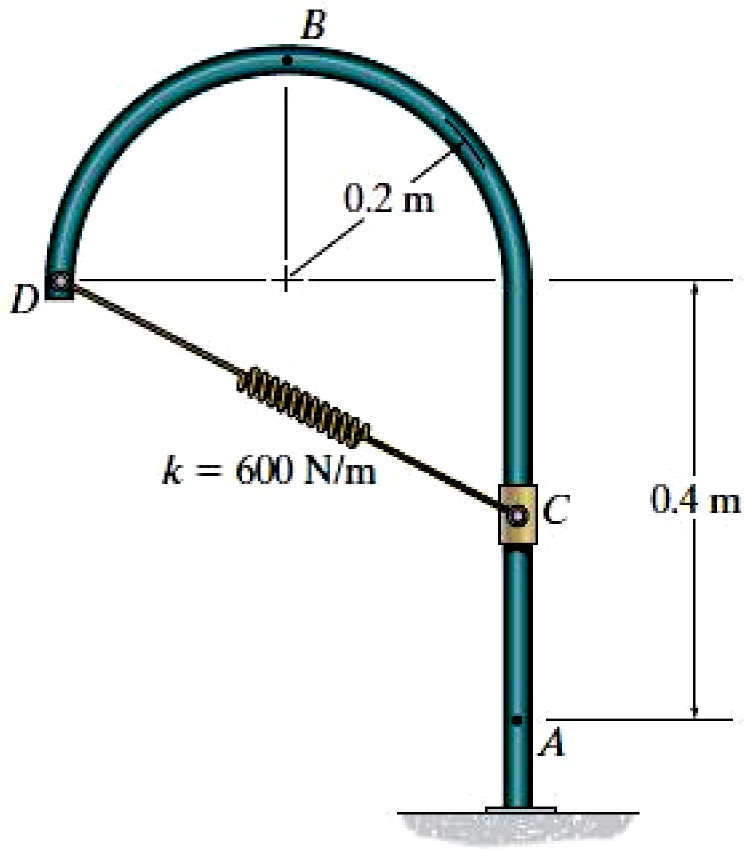
$$d = 0.6670 \text{ m}$$

$$(0.6670)^2 = (0.6)^2 + s^2$$

$s = 0.291$ m **Ans.**

p.215, 14-100

The 2-kg collar is released from rest at *A* and travels along the smooth vertical guide. Determine the speed of the collar when it reaches position *B*. Also, find the normal force exerted on the collar at this position. The spring has an unstretched length of 200 mm.

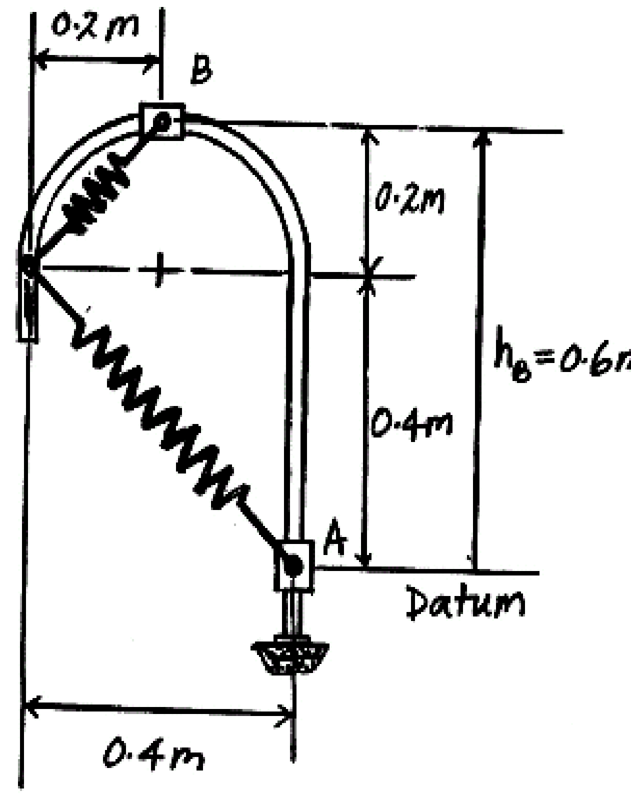


Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the collar at positions *A* and *B* are $(V_g)_A = mgh_A = 2(9.81)(0) = 0$ and $(V_g)_B = mgh_B = 2(9.81)(0.6) = 11.772$ J. When the collar is at positions *A* and *B*, the spring stretches $s_A = \sqrt{0.4^2 + 0.4^2} - 0.2 = 0.3657$ m and $s_B = \sqrt{0.2^2 + 0.2^2} - 0.2 = 0.08284$ m. Thus, the elastic potential energy of the spring when the collar is at these two positions are

$$(V_e)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(600)(0.3657^2) = 40.118 \text{ J}$$

and

$$(V_e)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(600)(0.08284^2) = 2.0589 \text{ J.}$$



(a)

Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + \left[(V_g)_A + (V_e)_A\right] = \frac{1}{2}mv_B{}^2 + \left[(V_g)_B + (V_e)_B\right]$$

$$0 + (0 + 40.118) = \frac{1}{2}(2)v_B{}^2 + (11.772 + 2.0589)$$

$$v_B = 5.127 \text{ m/s} = 5.13 \text{ m/s}$$ **Ans.**

Equation of Motion: When the collar is at position B, $\theta = \tan^{-1}\left(\frac{0.2}{0.2}\right) = 45°$ and $F_{sp} = ks_B = 600(0.08284) = 49.71$ N. Here,

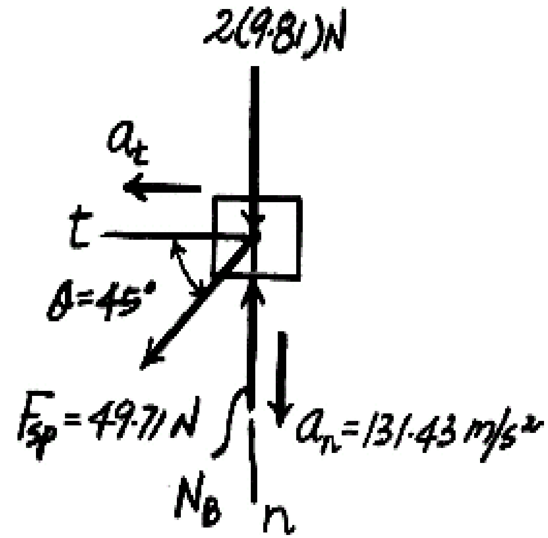
$$a_n = \frac{v^2}{\rho} = \frac{v_B{}^2}{0.2} = \frac{(5.127)^2}{0.2} = 131.43\,\text{m/s}^2.$$

By referring to the free-body diagram of the collar shown in Fig. b,

$$\Sigma F_n = ma_n; \qquad 2(9.81) + 49.71 \sin 45° - N_B = 2(131.43)$$

$$N_B = -208.09\,\text{N} = 208\,\text{N}\downarrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that $\mathbf{N}_B$ acts in the opposite sense to that shown on the free-body diagram.



(b)