Chap. 3
Equilibrium of a Particle

Chapter Outline

- Condition for the Equilibrium of a Particle
- The Free-Body Diagram
- Coplanar Systems
- Three-Dimensional Force Systems

Condition for the Equilibrium of a Particle

- $\sum \underline{\mathbf{F}} = 0$

  (充要條件)

  i.e. $m\underline{\mathbf{a}} = 0 \Rightarrow \underline{\mathbf{a}} = 0$

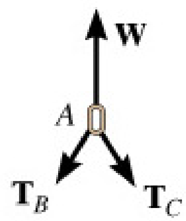
  (靜者恆靜,動者呈等速直線運動)

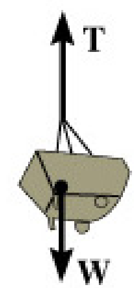
Free Body Diagram (FBD)

- 自由體圖

"The diagram of the particle (body) which represents it as being isolated or free from its surroundings ", it is necessary to show all the forces that act on the particle.

Free Body Diagram (FBD)



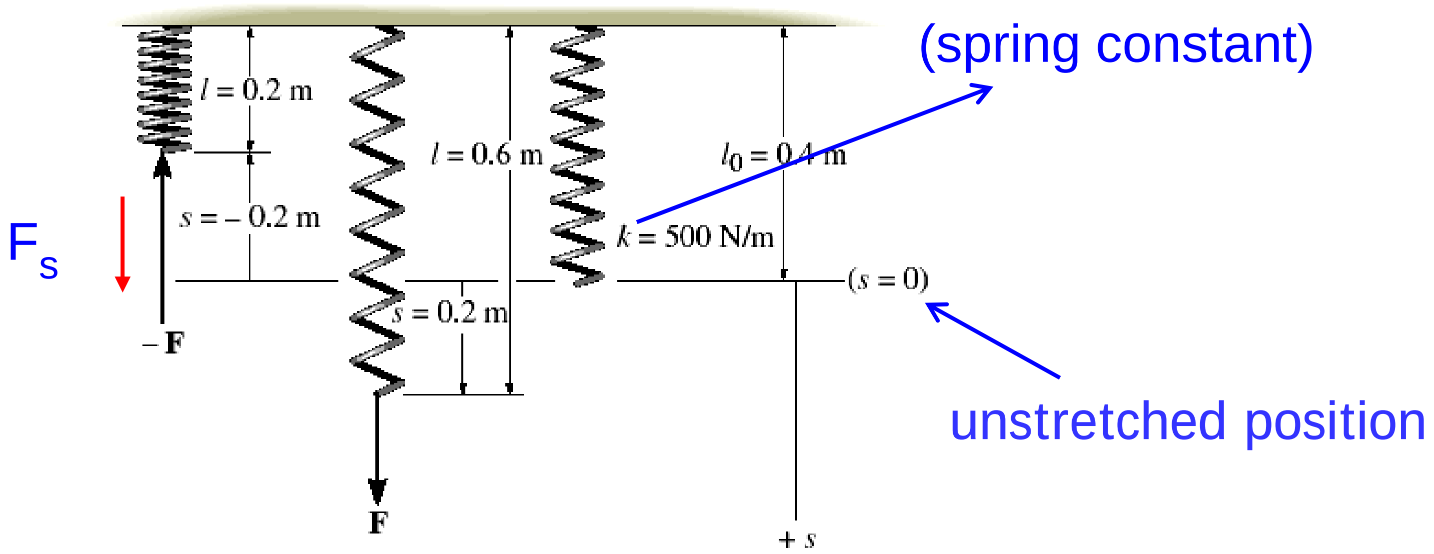


Free Body Diagram (FBD): Procedure

- Draw or sketch the outlined shape of the particle (body), and imagine the particle is isolated or cut free from its surroundings
- Indicate all the forces that act on the particle, including active and reactive forces
- Forces that are known should be labeled with their proper magnitudes and directions

Free Body Diagram (FBD)

- Springs

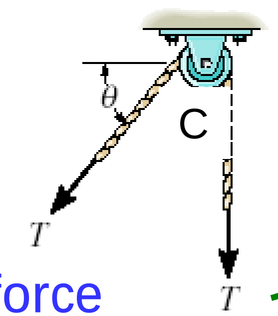


彈簧力 Fs = -ks

使彈簧變形的外力 F = ks

Free Body Diagram (FBD)

- Cables and Pulleys



Cable is in tension

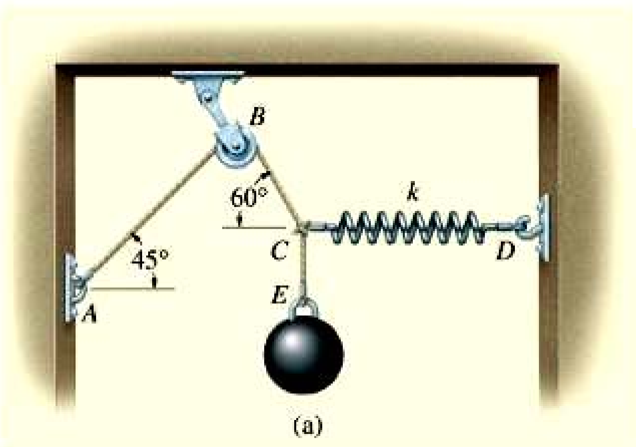
tension pulling force

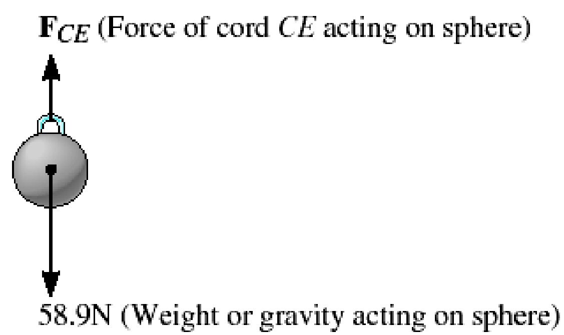
magnitude is constant
for the whole cable

往外為tension

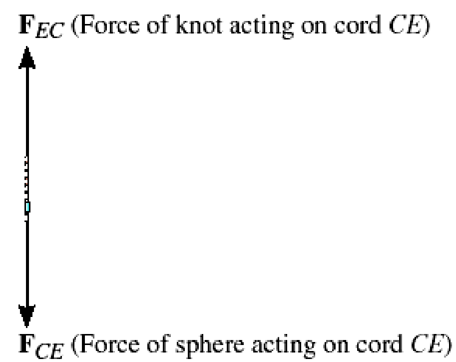
往內為compression

Ex. 3-1, draw the FBD of the sphere, cord *CE* and the knot at *C*

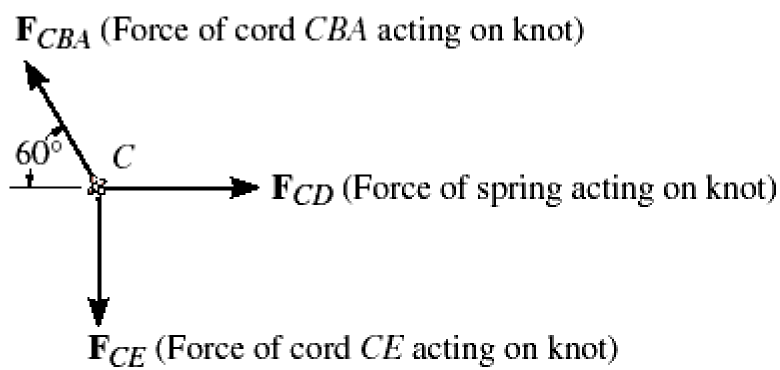




FBD of sphere



FBD of Cord *CE*



FBD of knot at C

Coplanar Force Systems 共面力

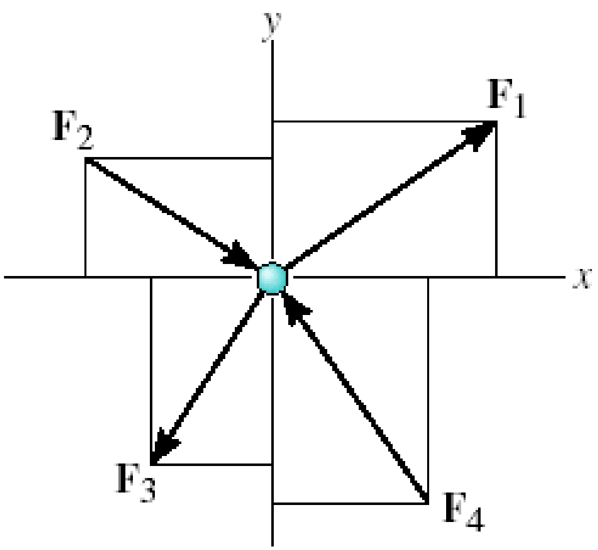


Figure 03.04

平衡的條件：

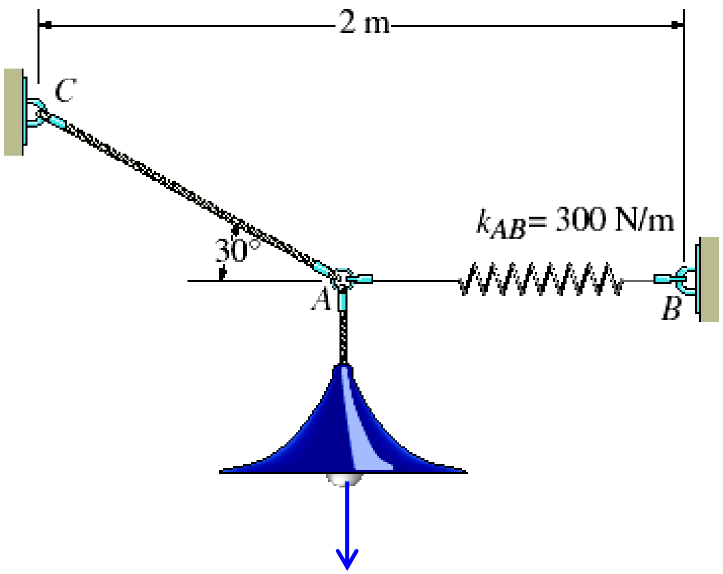
$$\sum \underline{F} = \sum F_x \underline{i} + \sum F_y \underline{j} = 0$$

$$\Rightarrow \quad \sum F_x = 0$$

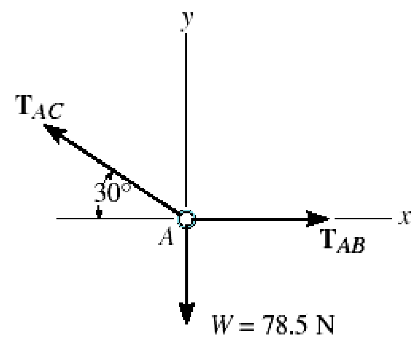
$$\sum F_y = 0$$

Ex. 3-4,

Determine the required length of cord *AC* so that the 8-kg lamp is suspended in the position shown. The undeformed length of spring is 0.4 m and has a stiffness of k_{AB} = 300 N/m.



8 kg = 78.48 N



$\Sigma F_x = 0: \; T_{AB} - T_{AC}\cos 30° = 0$

$\Sigma F_y = 0: \; T_{AC}\sin 30° - 78.48 = 0$

$\Rightarrow T_{AC} = 156.96 \text{ N}, \quad T_{AB} = 135.93 \text{ N}$

Spring on cable →

$S_{AB} = 135.93/300 = 0.4531 \text{ m}$

Deformation,cable on spring ←

$\therefore \text{spring length} = 0.4531 + 0.4 = 0.8531 \text{ m}$

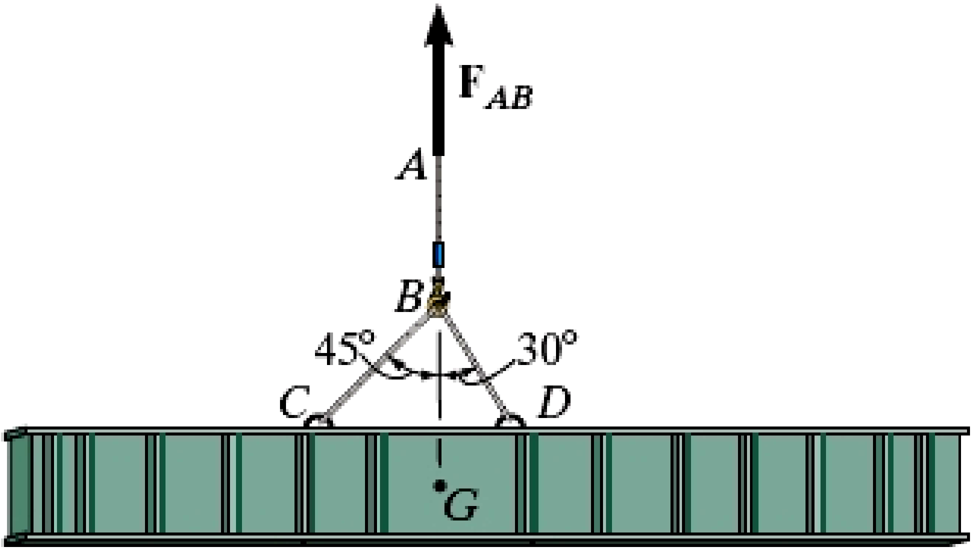
total horizontal length:

$2 = 0.8531 + \overline{AC} \cdot \cos 30°$

$\Rightarrow \quad \overline{AC} = 1.324 \text{ m}$

p.95, Problem 3-4

If cables *BD* and *BC* can withstand a maximum tensile force of 20-kN, determine the maximum mass of the girder that can be suspended from cable *AB* so that neither cable will fail. The center of mass of the girder is located at point G.



Equations of Equilibrium: The girder is suspended from cable AB. In order to meet the conditions of equilibrium the tensile force developed in cable AB must be equal to the weight of the girder. Thus,

$$F_{AB} = m(9.81) = 9.81m \qquad \textbf{Ans.}$$

Applying the equations of equilibrium along the x and y axes to the free - body diagram in Fig. (a),

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BD} \sin 30° - F_{BC} \sin 45° = 0$$

$$F_{BD} = 1.4142 F_{BC} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad 9.81m - F_{BD} \cos 30° - F_{BC} \cos 45° = 0 \qquad (2)$$

Since $F_{BD} > F_{BC}$, cable BD will break before cable BC. Substituting $F_{BD} = 20\ 000$ N into Eq. (1),

$$F_{BC} = 14\ 142.14 \text{ N}$$

Substituting this result into Eq. (2), yields

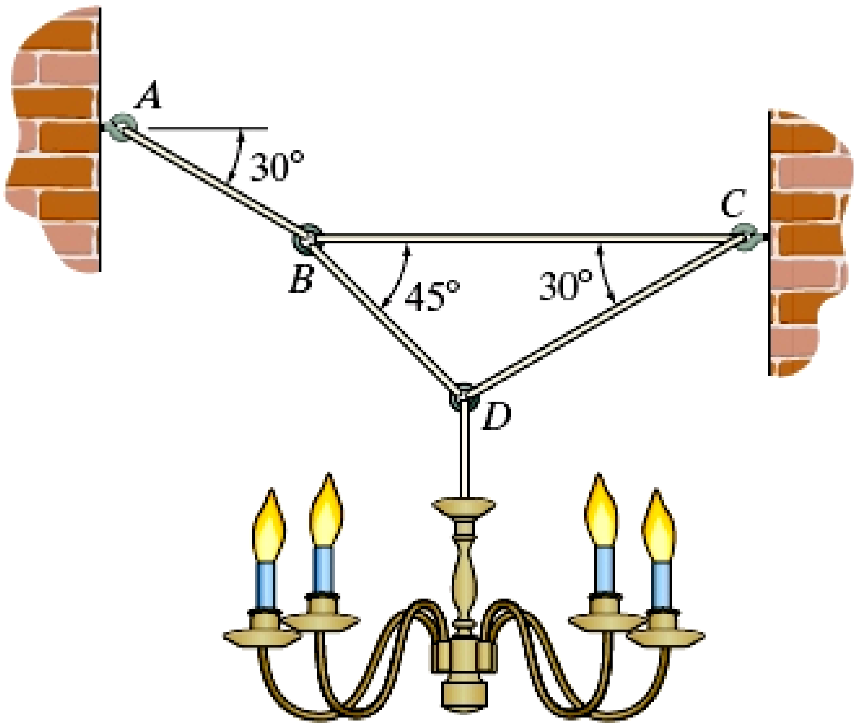
$$9.81m - 20\ 000 \cos 30° - 14\ 142.14 \cos 45° = 0$$

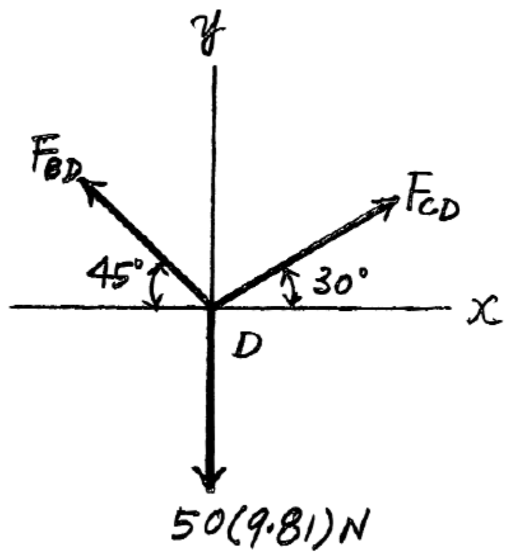
$$m = 2\ 785 \text{ kg} = 2.78 \text{ Mg} \qquad \textbf{Ans.}$$

significant number

p.97, Problem 3-20

Determine the tension developed in each wire used to support the 50-kg chandelier.





(a)

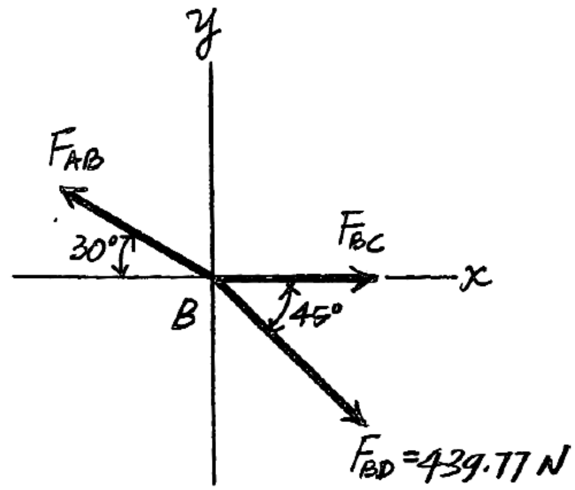
Equations of Equilibrium: First, we will apply the equations of equilibrium along the x and y axes to the free - body diagram of joint D shown in Fig. (a).

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{CD}\cos 30^\circ - F_{BD}\cos 45^\circ = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD}\sin 30^\circ + F_{BD}\sin 45^\circ - 50(9.81) = 0 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$F_{CD} = 359\,\text{N}$ $\qquad$ $F_{BD} = 439.77\,\text{N} = 440\,\text{N}$ $\qquad$ **Ans.**



(b)

Using the result $F_{BD} = 439.77$ N and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint B shown in Fig. (b),

$$+\uparrow \Sigma F_y = 0; \quad F_{AB}\sin 30° - 439.77\sin 45° = 0$$

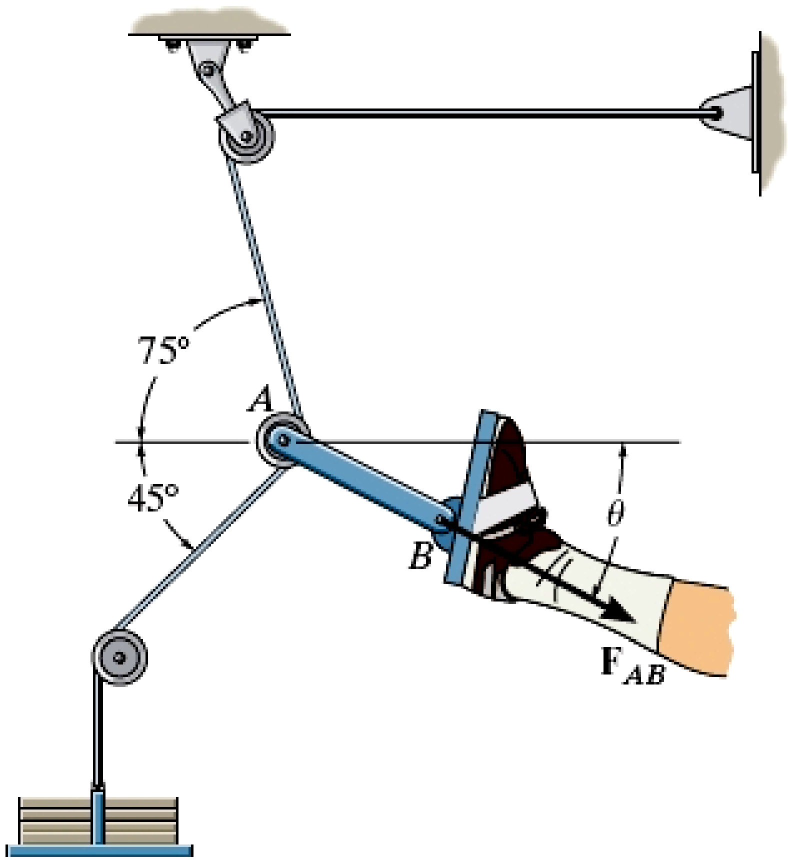
$$F_{AB} = 621.93\text{N} = 622\text{ N} \qquad \textbf{Ans.}$$

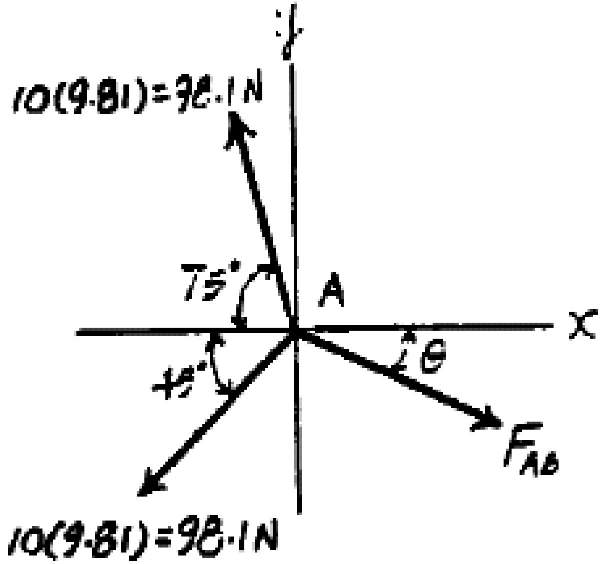
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{BC} + 439.77\cos 45° - 621.93\cos 30° = 0$$

$$F_{BC} = 228\text{N} \qquad \textbf{Ans.}$$

p.99, Problem 3-32

Determine the magnitude and direction θ of the equilibrium force F_{AB} exerted along link *AB* by the tractive apparatus shown. The suspended mass is 10-kg. Neglect the size of the pulley at *A*.





Free Body Diagram : The tension in the cord is the same throughout the cord, that is 10(9.81) = 9.81 N.

Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB}\cos\theta - 98.1\cos 75° - 98.1\cos 45° = 0$$

$$F_{AB}\cos\theta = 94.757 \qquad [1]$$

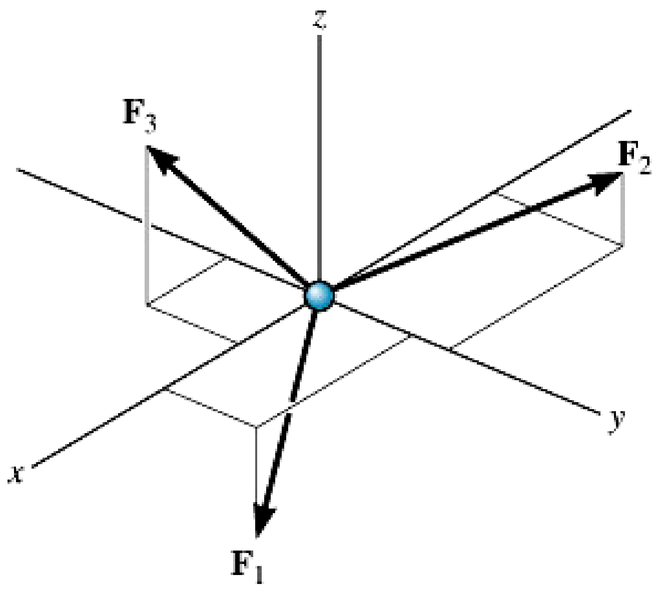
$$+\uparrow \Sigma F_y = 0; \qquad 98.1\sin 75° - 98.1\sin 45° - F_{AB}\sin\theta = 0$$

$$F_{AB}\sin\theta = 25.390 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$\theta = 15.0° \qquad F_{AB} = 98.1 \text{ N} \qquad \textbf{Ans}$$

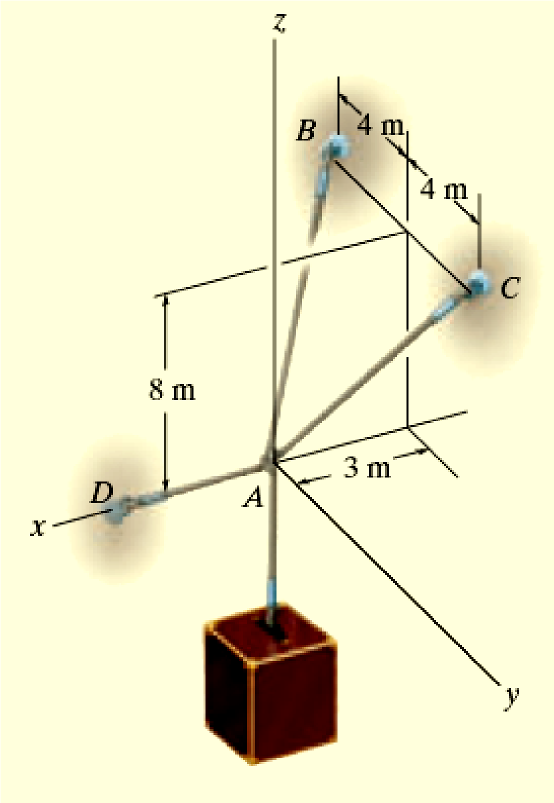
Three-Dimensional Force Systems



$$\Sigma \underline{F} = \Sigma F_x \underline{i} + \Sigma F_y \underline{j} + \Sigma F_z \underline{k} = 0$$

$$\Rightarrow \quad \begin{aligned} \sum Fx &= 0 \\ \sum Fy &= 0 \\ \sum Fz &= 0 \end{aligned}$$

Example 3.7
Determine the force developed in each cable used to support the 40-kN crate.



A(0, 0, 0), B(-3, -4, 8), C(-3, 4, 8)

FBD at Point A

To expose all three unknown forces in the cables.

Equations of Equilibrium

Expressing each forces in Cartesian vectors,

$\mathbf{F}_B = F_B(\mathbf{r}_B / r_B)$

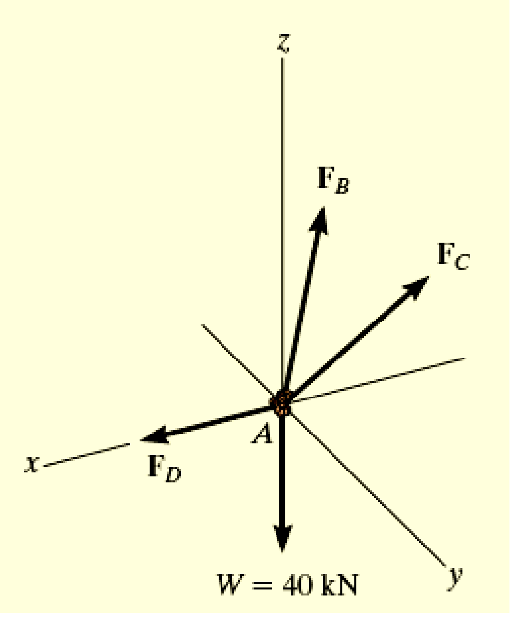
$= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$

$\mathbf{F}_C = F_C(\mathbf{r}_C / r_C)$

$= -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$

$\mathbf{F}_D = F_D\mathbf{i}$

$\mathbf{W} = -40\mathbf{k}$



For equilibrium,

$$\sum \mathbf{F} = 0; \qquad \mathbf{F}_{\mathrm{B}} + \mathbf{F}_{\mathrm{C}} + \mathbf{F}_{\mathrm{D}} + \mathbf{W} = 0$$

$$-0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k} - 0.318F_{\mathrm{C}}\mathbf{i} + 0.424F_{\mathrm{C}}\mathbf{j} + 0.848F_{\mathrm{C}}\mathbf{k} + F_{\mathrm{D}}\mathbf{i} - 40\mathbf{k} = 0$$

$$\sum F_x = 0; \qquad -0.318F_B - 0.318F_{\mathrm{C}} + F_{\mathrm{D}} = 0$$

$$\sum F_y = 0; \qquad -0.424F_B + 0.424F_{\mathrm{C}} = 0$$

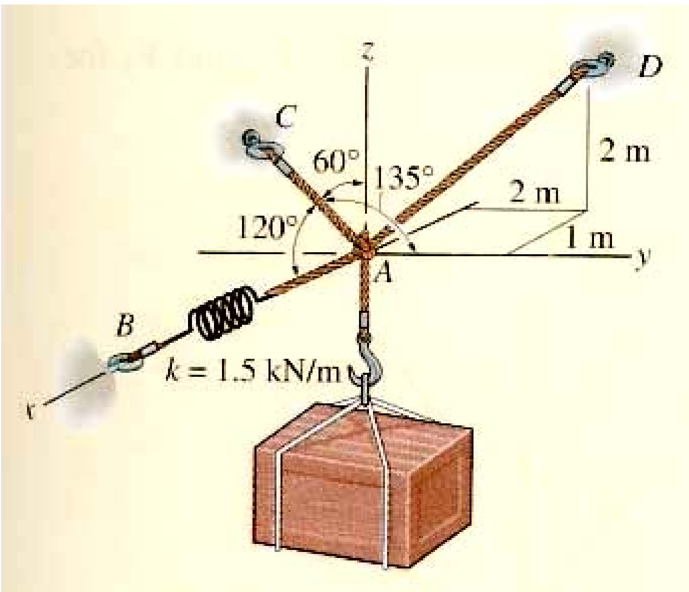
$$\sum F_z = 0; \qquad 0.848F_B + 0.848F_{\mathrm{C}} - 40 = 0$$

Solving,

$F_B = F_{\mathrm{C}} = 23.6$ kN

$F_{\mathrm{D}} = 15.0$ kN

Ex. 3-8,
Determine the tension in each cord and the stretch of the spring.



$$\overrightarrow{AD} = (-1,2,2) \quad AD = \sqrt{(-1)^2 + 2^2 + 2^2}$$

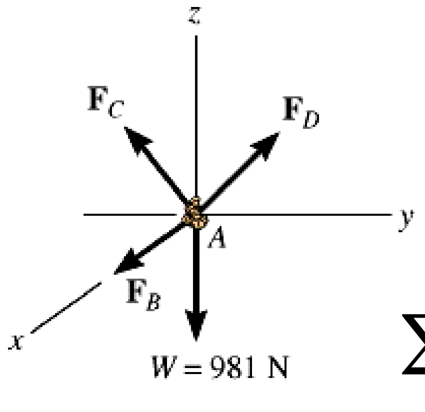
$$\underline{u}_{AD} = \frac{1}{3}(-1,2,2)$$

$$\underline{F}_D = \frac{F_D}{3}(-1,2,2)$$

$$\underline{F}_B = F_B \underline{i}$$

$$\underline{F}_C = F_C(\cos 120°, \cos 135°, \cos 60°)$$

$$\underline{W} = -981 \text{ kN } \underline{k}$$



(b)

$$\Sigma \underline{F} = 0:$$

$$\underline{F}_B + \underline{F}_C + \underline{F}_D + \underline{W} = 0$$

$$\begin{cases} \Sigma F_x = 0: F_B + F_C \cos 120° - \frac{1}{3} F_D = 0 \\ \Sigma F_y = 0: F_C \cos 135° + \frac{2}{3} F_D = 0 \\ \Sigma F_Z = 0: F_C \cos 60° + \frac{2}{3} F_D - 981 = 0 \end{cases}$$

$\therefore$ F_B = 694 N ， F_C = 813N , F_D = 862 N 又 $F_B = ks$

$\Rightarrow 693.7 = 1500s \Longrightarrow$ s = 0.462 m

p.110, Problem 3-56

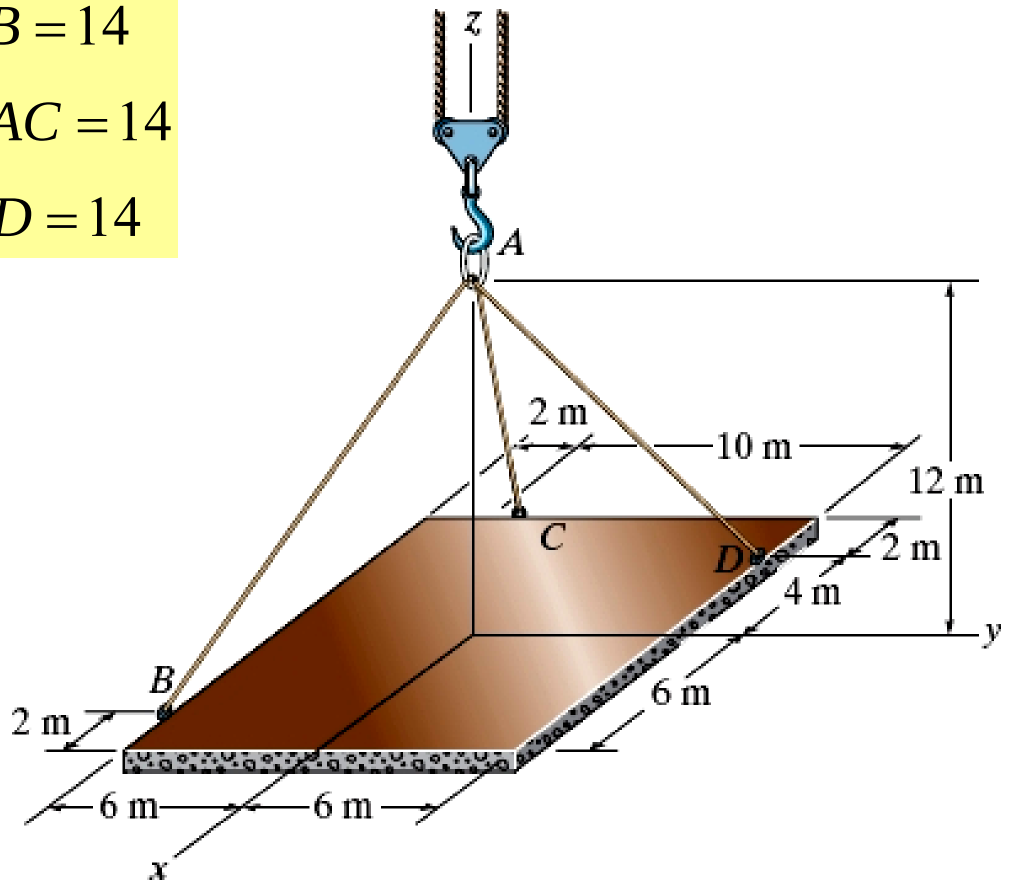
The ends of the three cables are attached to a ring at *A* and to the edge of a uniform 150-kg plate. Determine the tension in each of the cables for equilibrium.

A(0, 0, 12), B(4, -6, 0), C(-6, -4, 0), D(-4, 6, 0)

$$\overrightarrow{AB} = (4, -6, -12),\ AB = 14$$

$$\overrightarrow{AC} = (-6, -4, -12),\ AC = 14$$

$$\overrightarrow{AD} = (-4, 6, -12),\ AD = 14$$

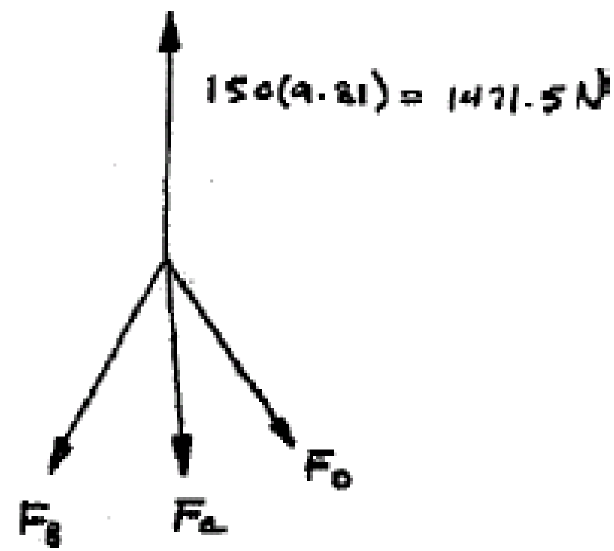


$$\mathbf{P} = 150(9.81)\,\mathbf{k} = 1471.5\,\mathbf{k}$$

$$\mathbf{F}_B = \frac{4}{14}F_B\,\mathbf{i} - \frac{6}{14}F_B\,\mathbf{j} - \frac{12}{14}F_B\,\mathbf{k}$$

$$\mathbf{F}_C = -\frac{6}{14}F_C\,\mathbf{i} - \frac{4}{14}F_C\,\mathbf{j} - \frac{12}{14}F_C\,\mathbf{k}$$

$$\mathbf{F}_D = -\frac{4}{14}F_D\,\mathbf{i} + \frac{6}{14}F_D\,\mathbf{j} - \frac{12}{14}F_D\,\mathbf{k}$$



$$\Sigma F_x = 0; \qquad \frac{4}{14} F_B - \frac{6}{14} F_C - \frac{4}{14} F_D = 0$$

$$\Sigma F_y = 0; \qquad -\frac{6}{14} F_B - \frac{4}{14} F_C + \frac{6}{14} F_D = 0$$

$$\Sigma F_z = 0; \qquad -\frac{12}{14} F_B - \frac{12}{14} F_C - \frac{12}{14} F_D + 1471.5 = 0$$

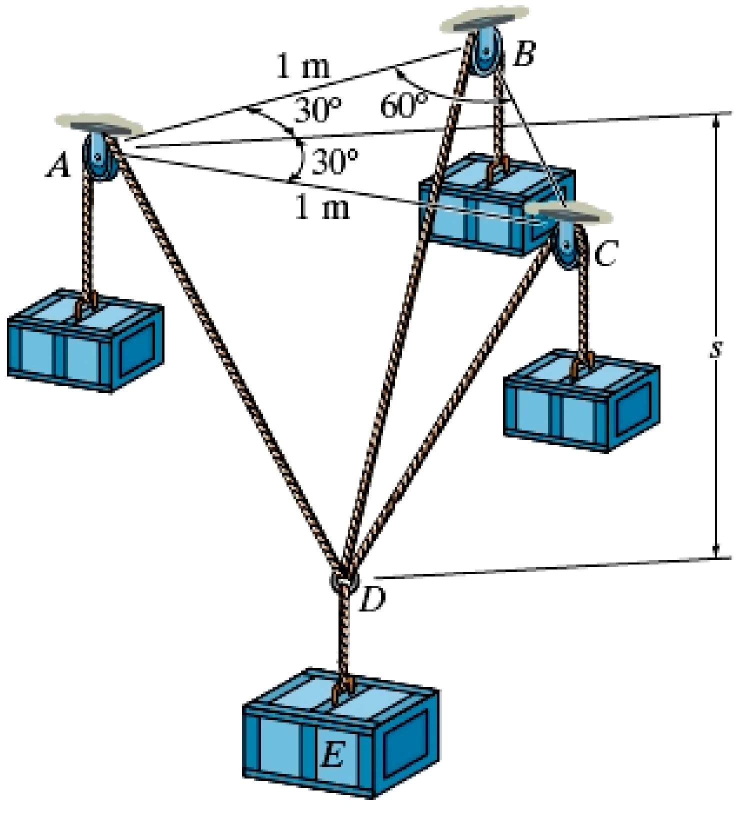
$F_B = 858\text{ N}$ Ans

$F_C = 0$ Ans

$F_D = 858\text{ N}$ Ans

p.112, Problem 3-68

The three outer blocks each have a mass of 2 kg, and the central block *E* has a mass of 3 kg. Determine the sag *s* for equilibrium of the system.



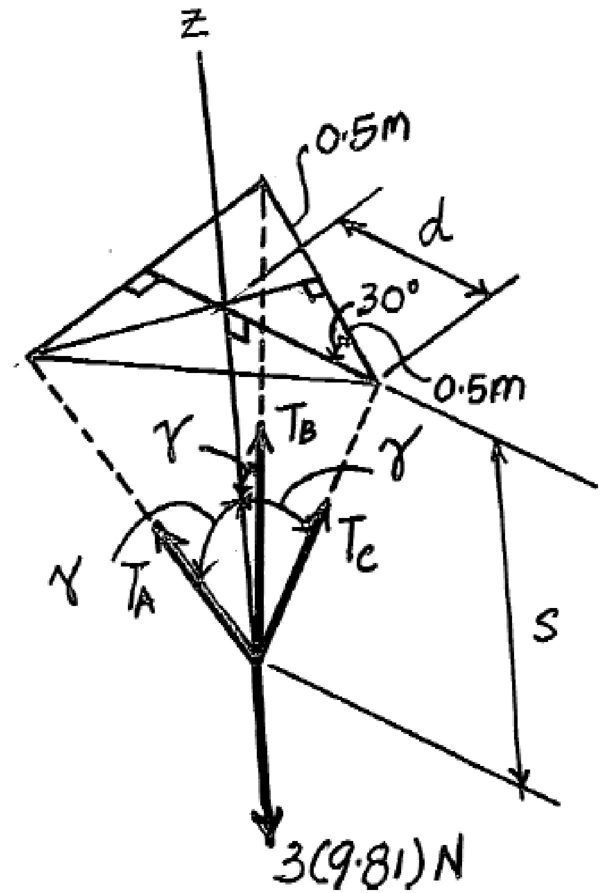
$$T_A = T_B = T_C = 2\,(9.81)$$

$$\Sigma F_z = 0; \quad 3\,(2\,(9.81))\cos\gamma - 3\,(9.81) = 0$$

$$\cos\gamma = 0.5; \quad \gamma = 60^\circ$$

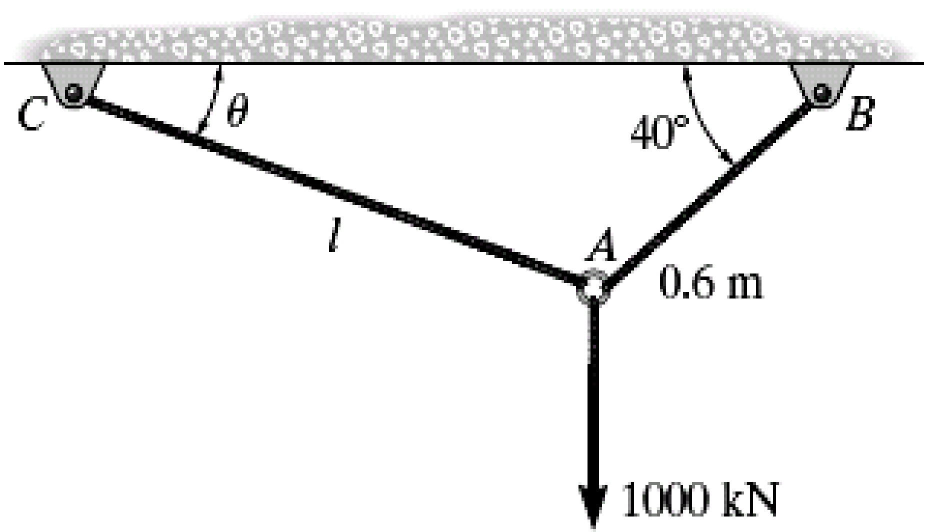
$$d = \frac{0.5}{\cos 30^\circ} = 0.577\ \text{m}$$

$$s = \frac{0.577}{\tan 60^\circ} = 0.333\ \text{m} = 333\ \text{mm} \qquad \textbf{Ans}$$



p.115, 3-76.

The ring of negligible size is subjected to a vertical force of 1000-N. Determine the longest length *l* of cord *AC* such that the tension acting in *AC* is 800-N. Also, what is the force acting in cord *AB*? *Hint:* Use the equilibrium condition to determine the required angle θ for attachment, then determine *l* using trigonometry applied to ΔABC.



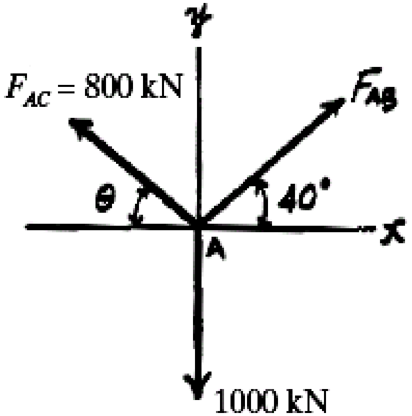
$(-\sin 40°)\times[1] + \cos 40°)\times[2]$

$\Rightarrow 800 \sin(40°+\theta) = 1000 \cos 40°$

Equations of Equilibrium :

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{AB} \cos 40° - 800 \cos \theta = 0$ [1]

$+\uparrow\Sigma F_y = 0; \quad F_{AB} \sin 40° + 800 \sin \theta - 1000 = 0$ [2]



Solving Eqs. [1] and [2] yields

$\theta = 33.25°$

$F_{AB} = 873.4$ N **Ans**

Geometry : Applying law of sines, we have

$$\frac{l}{\sin 40°} = \frac{0.6}{\sin 33.25°}$$

$l = 0.703$ m **Ans**

End of Chapter 3