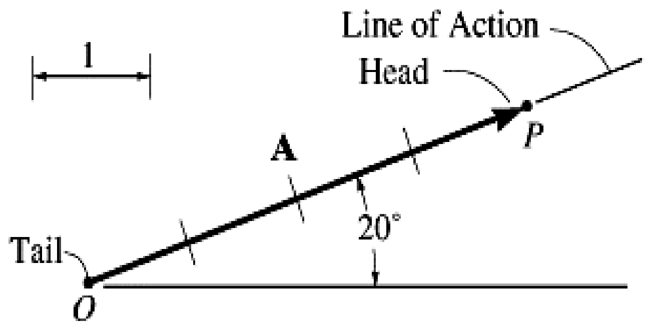
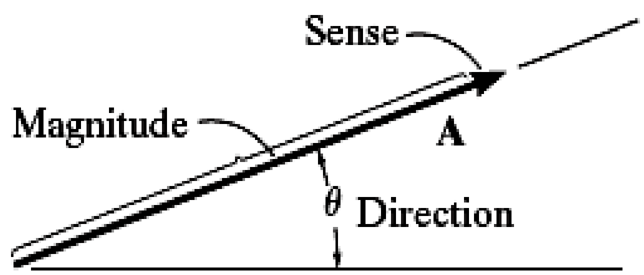
Chap. 2 Force Vectors

Chapter Outline

- Scalars and Vectors
- Vector Operations
- Vector Addition of Forces
- Addition of a System of Coplanar Forces
- Cartesian Vectors
- Addition and Subtraction of Cartesian Vectors
- Position Vectors
- Force Vector Directed along a Line
- Dot Product

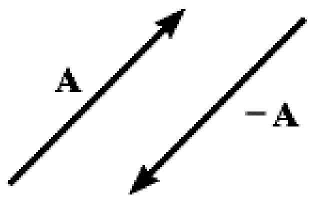
Scalar and Vectors



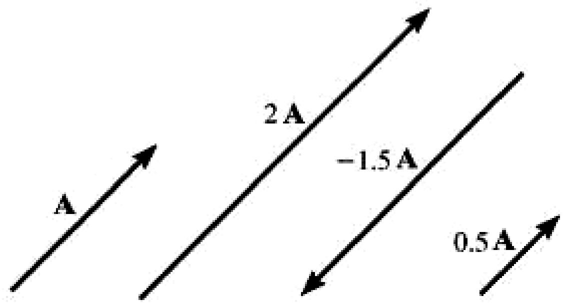


Vector Operations

- Multiplication and Division by a Scalar

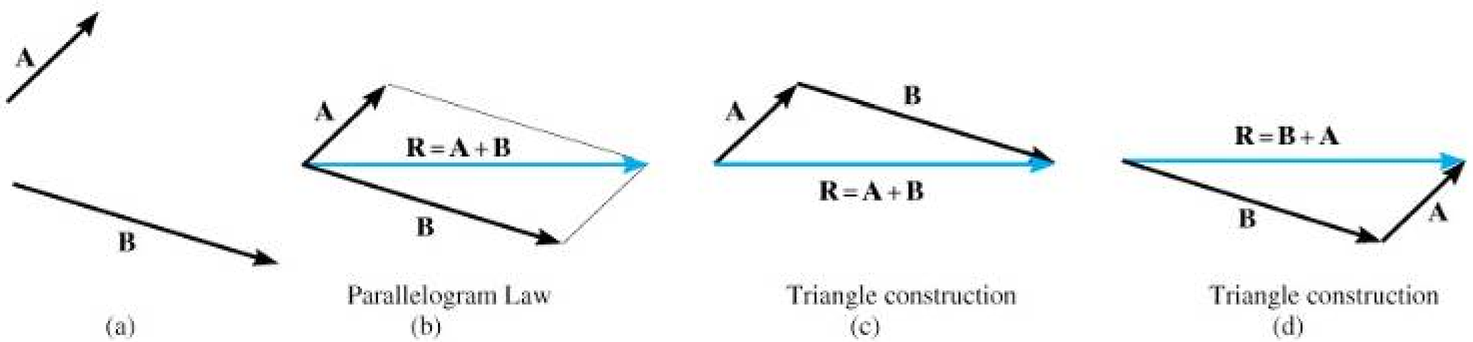


Vector **A** and its negative counterpart



Scalar Multiplication and Division

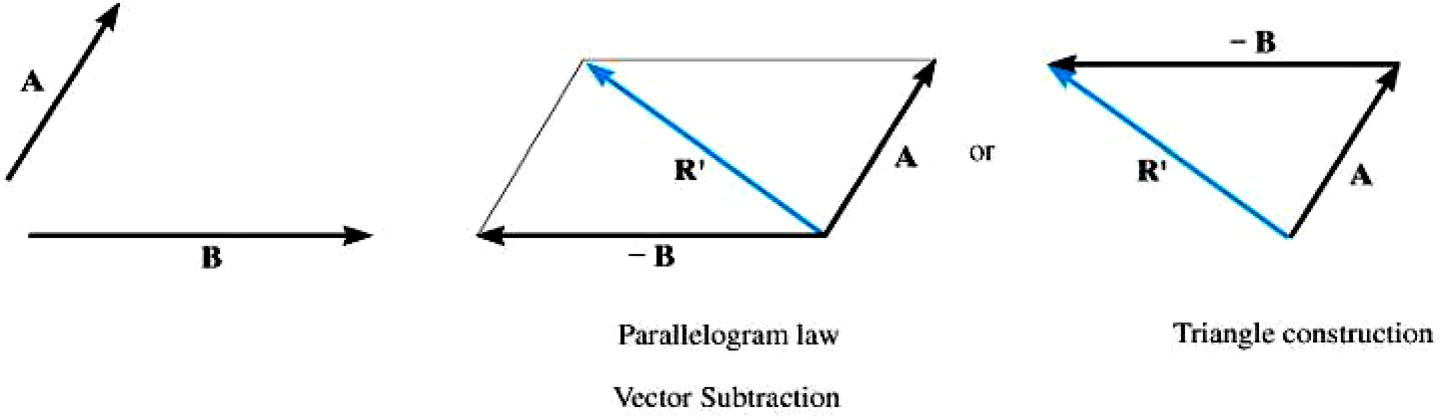
- Vector Addition



Vector Addition

Vector Operations

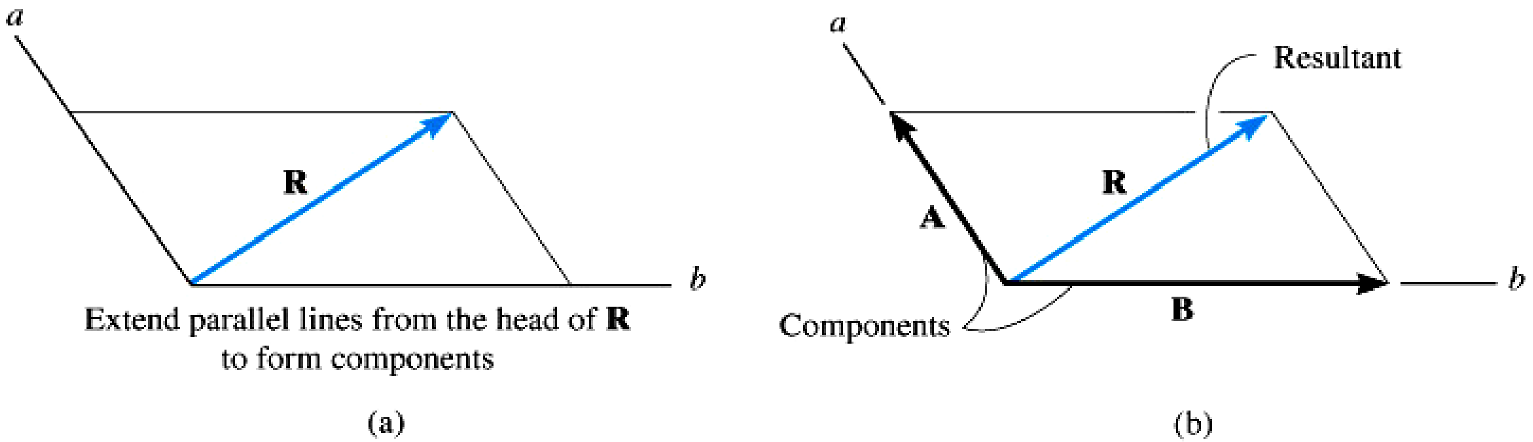
- Vector Subtraction



*** draw the vector from the head of B to the head of A**

Vector Operations

- Resolution of a Vector

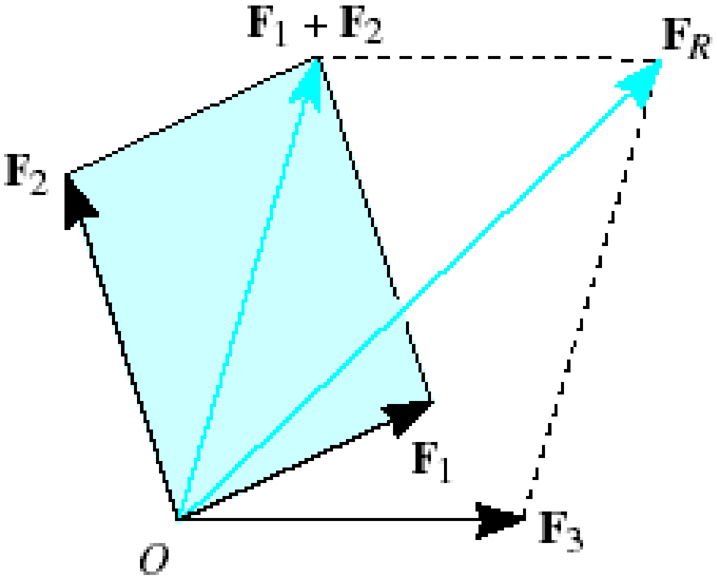


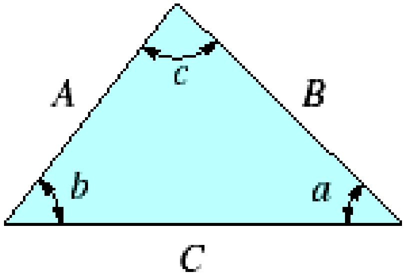
Resolution of a vector

1. 已知R和A求B
2. 已知R和A、B的作用線

Vector Operations

- Vector Addition of Forces





Sine law:

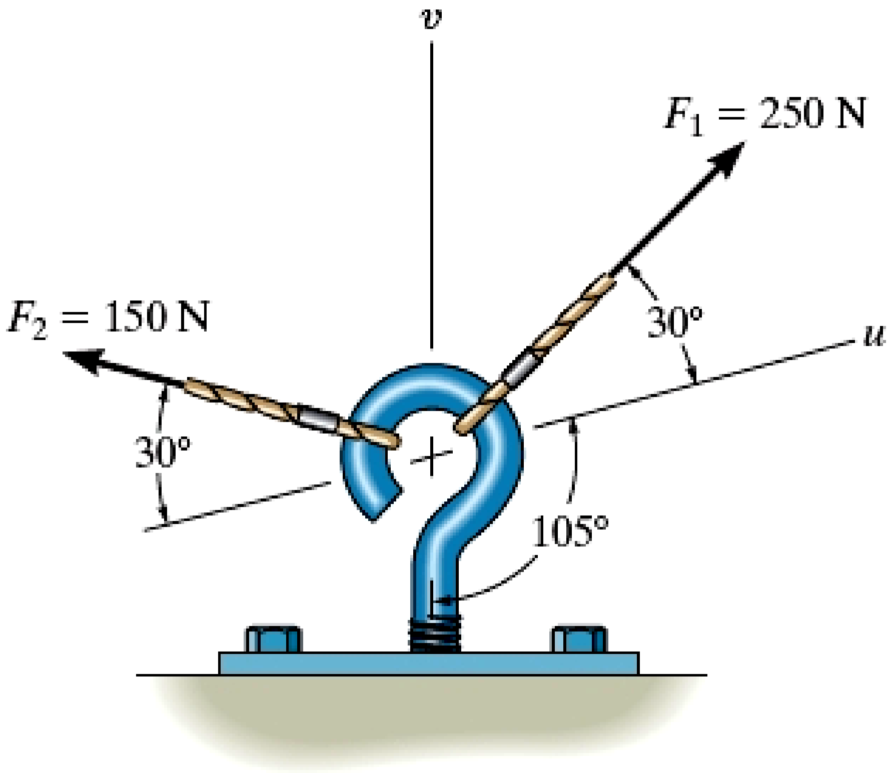
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2\,A\,B\cos c}$$

p.29, Problem 2-17

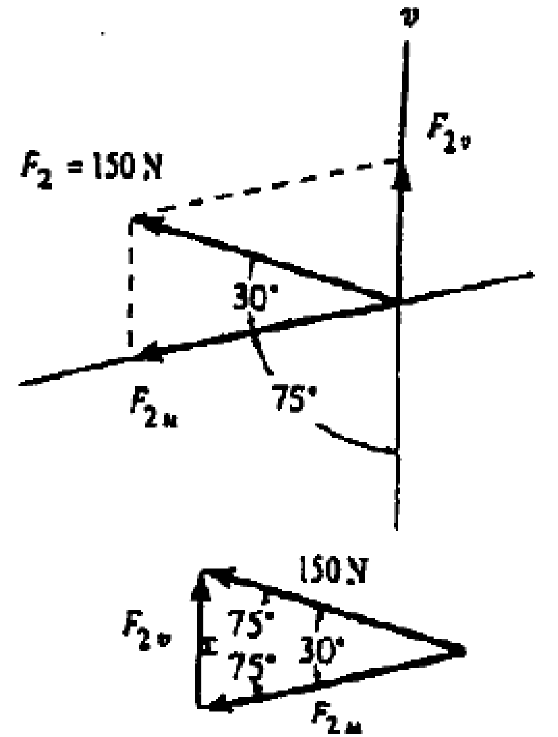
Resolve the force $\mathbf{F_2}$ into components acting along the *u* and *v* axes and determine the magnitudes of the components.



Sine law. :

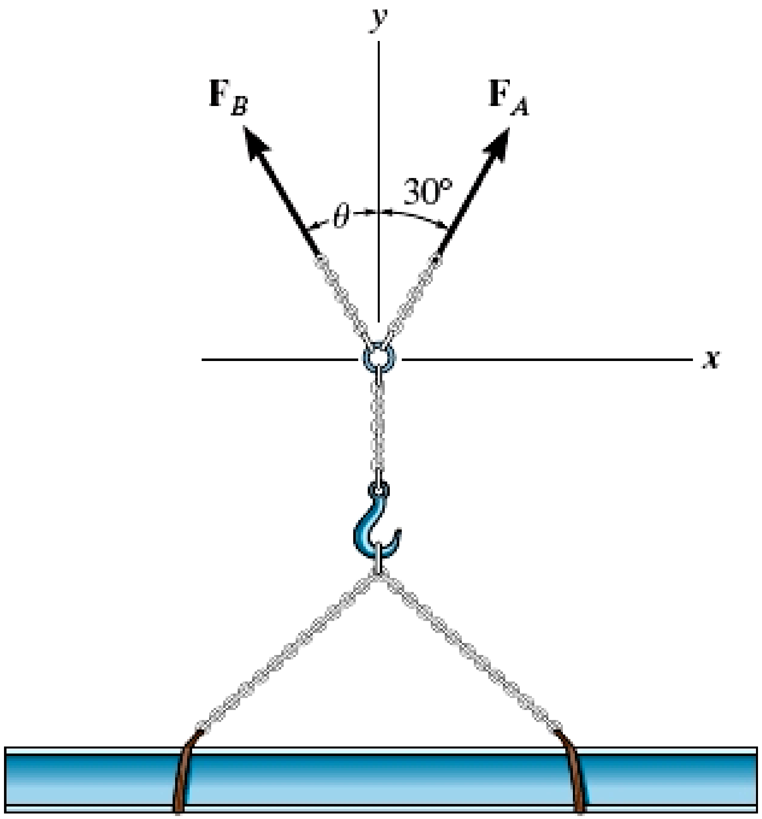
$$\frac{F_{2v}}{\sin 30^\circ} = \frac{150}{\sin 75^\circ} \qquad F_{2v} = 77.6\ \text{N} \qquad \text{Ans}$$

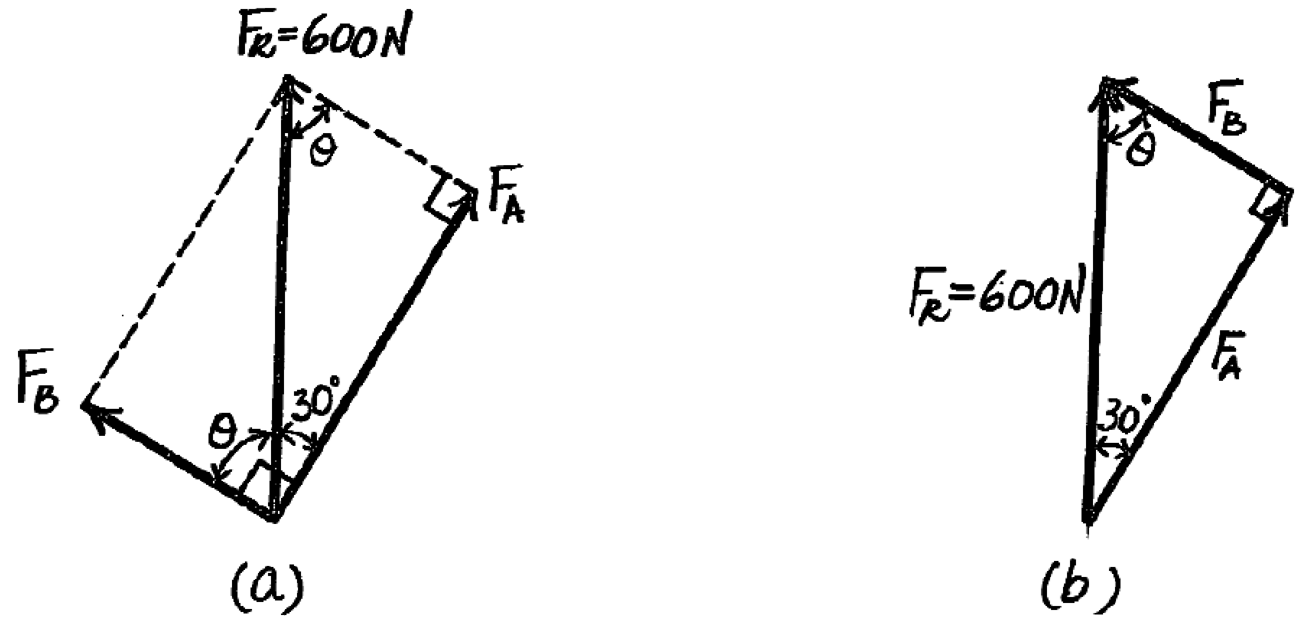
$$\frac{F_{2u}}{\sin 75^\circ} = \frac{150}{\sin 75^\circ} \qquad F_{2u} = 150\ \text{N} \qquad \text{Ans}$$



p.31, Problem 2-29

The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive *y* axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain and the angle θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a minimum. $\mathbf{F}_A$ acts at 30° from the *y* axis as shown.





For minimum F_B, require

$\theta = 60°$ Ans

$F_A = 600 \cos 30° = 520$ N Ans

$F_B = 600 \sin 30° = 300$ N Ans

Addition of a System of Coplanar Forces

- Cartesian Vector

  $\underline{\mathbf{F}} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$

- Resultants

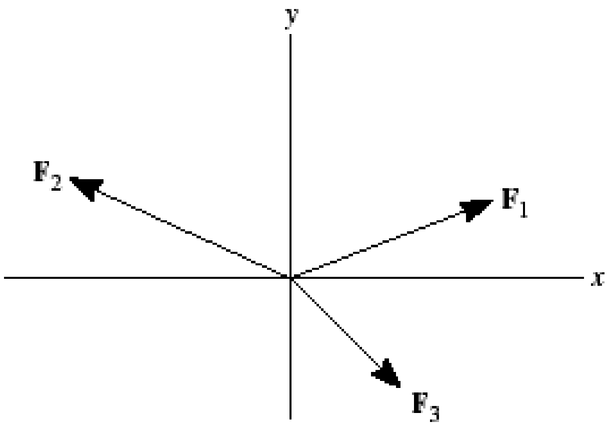


Figure 02.16-1(a)

$$\overrightarrow{F_R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3}$$
$$= (F_{1x} + F_{2x} + F_{3x})\,i$$
$$+ (F_{1y} + F_{2y} + F_{3y})\,j$$
$$= (F_{Rx})\,i + (F_{Ry})\,j$$

Addition of a System of Coplanar Forces

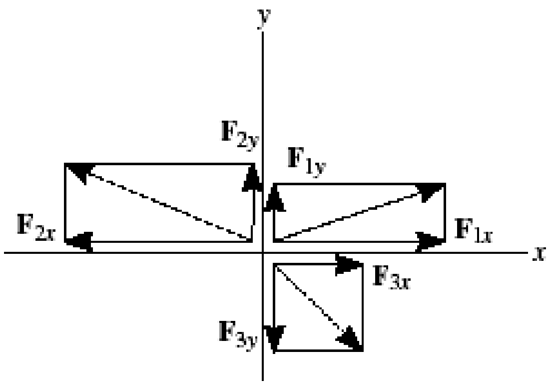


Figure 02.16-1(b)

$$F_{Rx} = \Sigma F_x$$

$$F_{Ry} = \Sigma F_y$$

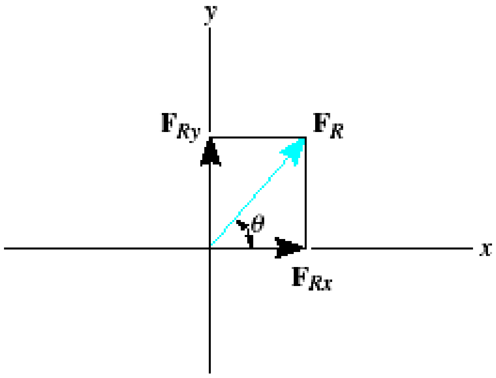


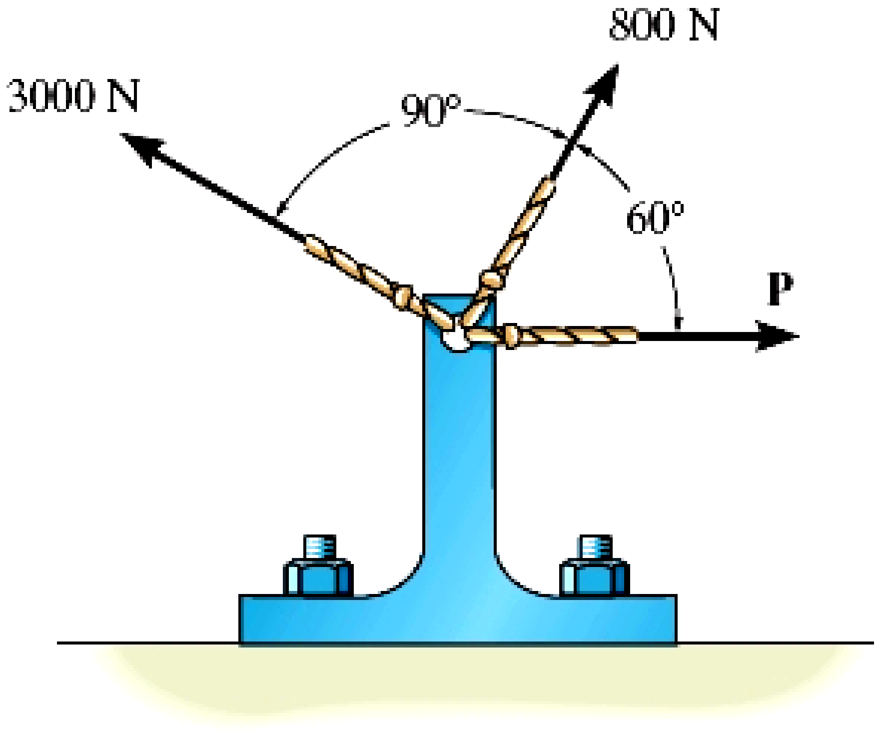
Figure 02.16-1(c)

$$F_R = \sqrt{F_{Rx}^{\ 2} + F_{Ry}^{\ 2}}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

p.41, Problem 2-50

The three forces are applied to the bracket. Determine the range of values for the magnitude of force **P** so that the resultant of the three forces does not exceed 2400 N.



$\rightarrow F_{Rx} = \Sigma F_x$: $F_{Rx} = P + 800 \cos 60° - 3000 \cos 30° = P - 2198.08$

$+\uparrow F_{Ry} = \Sigma F_y$: $F_{Ry} = 800 \sin 60° + 3000 \sin 30° = 2192.82$

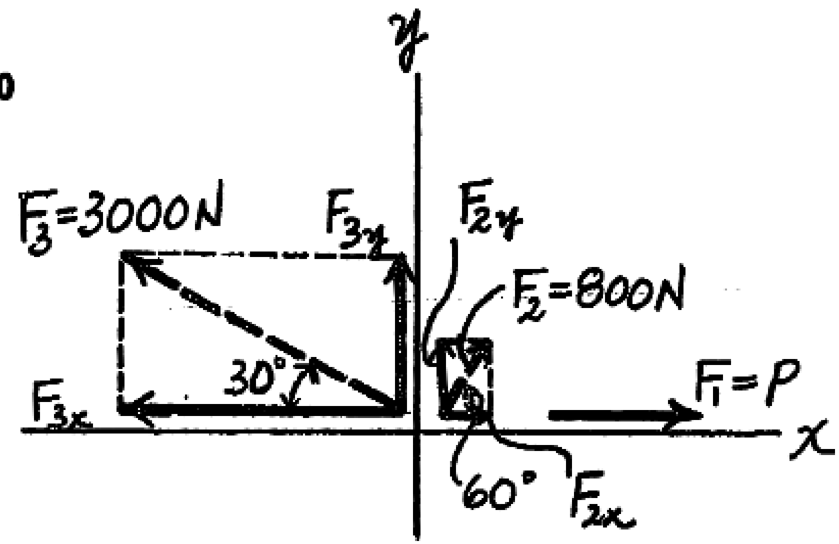
$$F_R = \sqrt{(P - 2198.08)^2 + (2192.82)^2} \leq 2400$$

$$(P - 2198.08)^2 + (2192.82)^2 \leq (2400)^2$$

$$|(P - 2198.08)| \leq 975.47$$

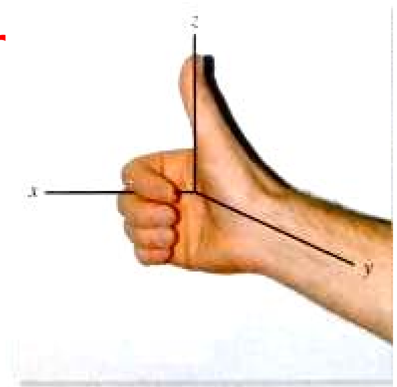
$$-975.47 \leq P - 2198.08 \leq 975.47$$

$$1222.6\text{ N} \leq P \leq 3173.5\text{ N}$$



Cartesian Vectors

- Right-Handed Coordinate System
- Rectangular Components of a Vector



Right-handed coordinate system.

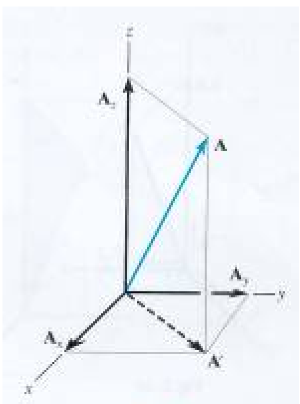


Fig. 2–21

$$\mathbf{A}=\mathbf{A}_x+\mathbf{A}_y+\mathbf{A}_z$$

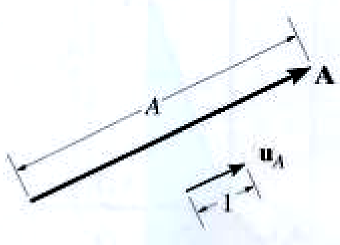


Fig. 2–22

- Unit Vector $\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{\mathbf{A}}{|\mathbf{A}|}$
- Cartesian Unit Vector **i , j , k**

Cartesian Vectors

- Cartesian Vector Representation

A = A**x** **i** + A**y** **j** + A**z** **k**

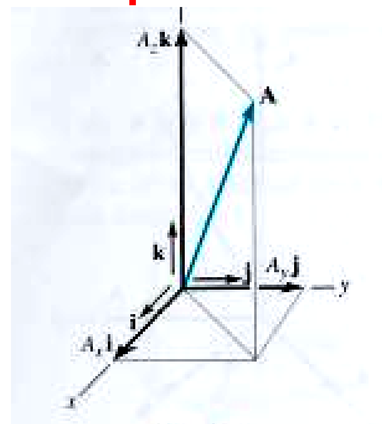


Fig. 2–24

- Magnitude

$$|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{A'^2 + A_z^2}$$

A在xy平面上的投影

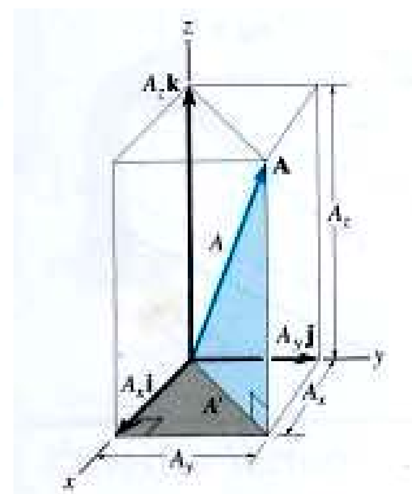


Fig. 2–25

Cartesian Vectors

- **Direction**

  α, β, γ 及方向餘弦(direction cosine)

  $$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

- **單位向量 (unit vector)**

  $$\underline{u}_A = \frac{\underline{A}}{A} = \frac{A_x}{A} i + \frac{A_y}{A} j + \frac{A_z}{A} k$$

  $$= \cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k$$

  又 $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

  $$\underline{A} = A\underline{u_A}$$

  $$= A\cos\alpha\, i + A\cos\beta\, j + A\cos\gamma\, k$$

  $$= A_x\, i + A_y\, j + A_z\, k$$

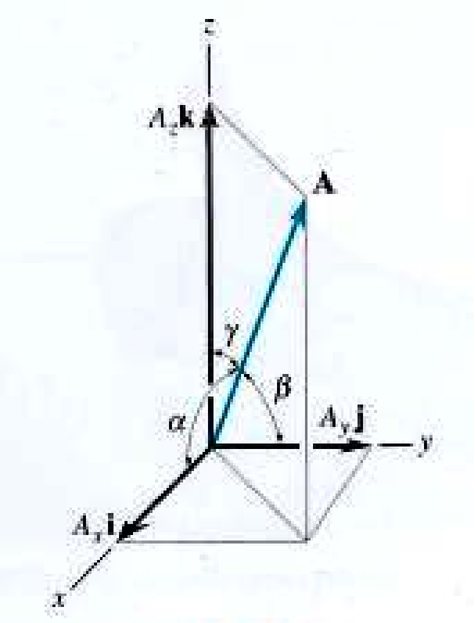


Fig. 2–26

Cartesian Vectors

- Addition and Subtraction of Cartesian Vectors

R = **A**+**B** = (Ax+Bx)**i** + (Ay+By)**j** + (Az+Bz)**k**

R'= **A**-**B** = (Ax - Bx)**i** + (Ay - By)**j** + (Az - Bz)**k**

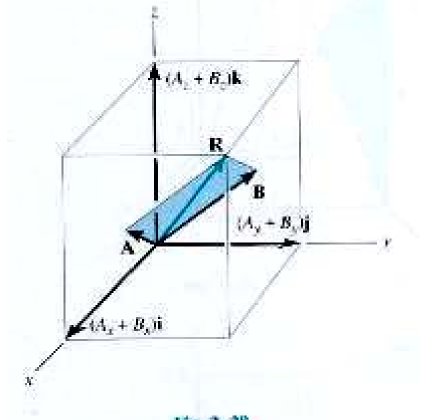


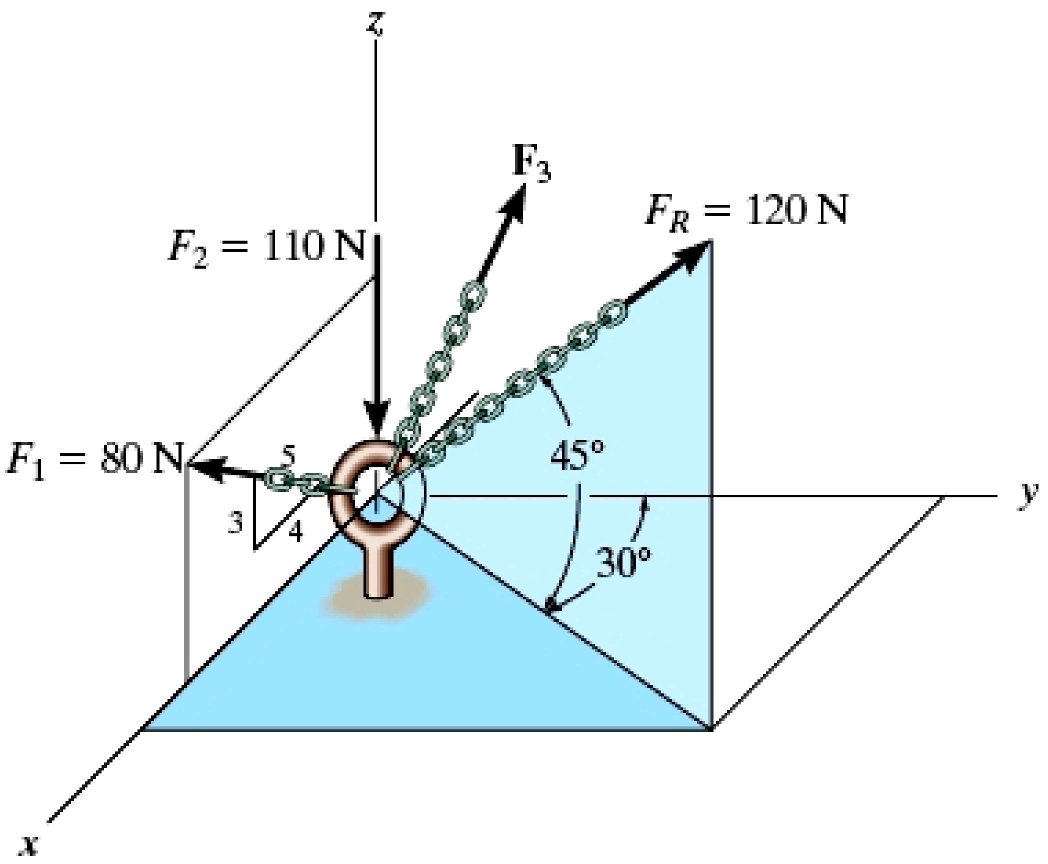
Fig. 2–28

- Concurrent Force Systems

$$\underline{F_R} = \Sigma \underline{F} = \Sigma F_x i + \Sigma F_y j + \Sigma F_z k$$

p.55, Problem 2-84

Determine the coordinate direction angles of $\mathbf{F_1}$ and $\mathbf{F_R}$.



Unit Vector of $\mathbf{F}_1$ and $\mathbf{F}_R$:

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{u}_R = \cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k}$$
$$= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

Thus, the coordinate direction angles $\mathbf{F}_1$ and $\mathbf{F}_R$ are

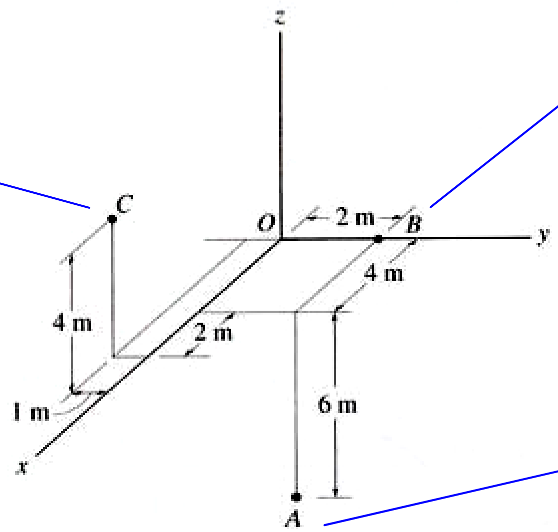
$\cos \alpha_{F_1} = 0.8$	$\alpha_{F_1} = 36.9^\circ$	**Ans**
$\cos \beta_{F_1} = 0$	$\beta_{F_1} = 90.0^\circ$	**Ans**
$\cos \gamma_{F_1} = 0.6$	$\gamma_{F_1} = 53.1^\circ$	**Ans**
$\cos \alpha_R = 0.3536$	$\alpha_R = 69.3^\circ$	**Ans**
$\cos \beta_R = 0.6124$	$\beta_R = 52.2^\circ$	**Ans**
$\cos \gamma_R = 0.7071$	$\gamma_R = 45.0^\circ$	**Ans**

Position Vectors

- x, y, z Coordinates 座標 (位置向量)

(6,-1,4)

(0,2,0)

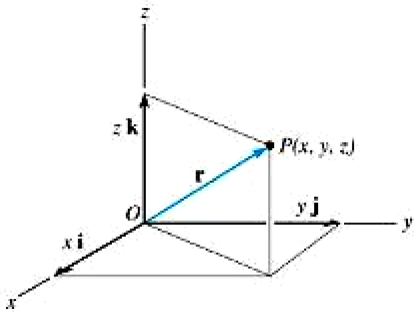


(4,2,-6)

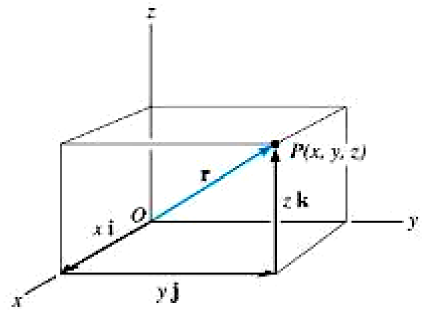
Position Vectors

- Position Vector 位置向量

  相對於固定點0的向量



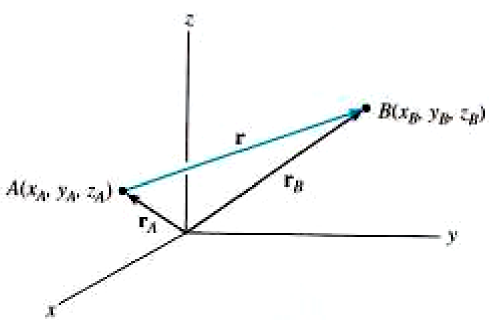
(a)



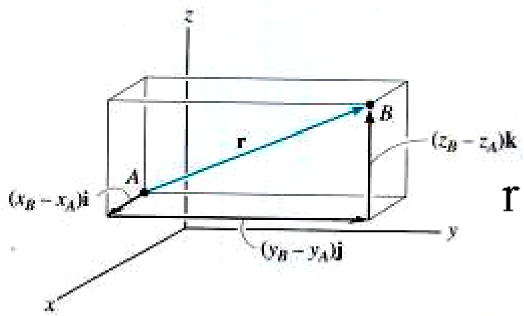
(b)

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

- Relative Position Vector



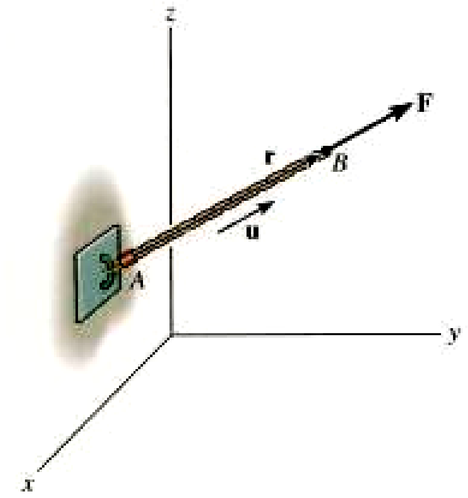
(a)



(b)

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = (x_B - x_A, y_B - y_A, z_B - z_A)$$

Force Vector Directed along a Line

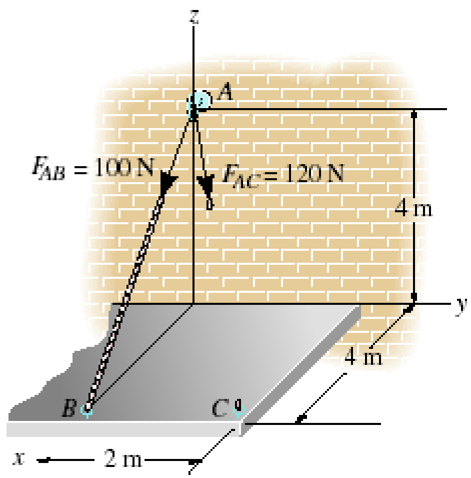


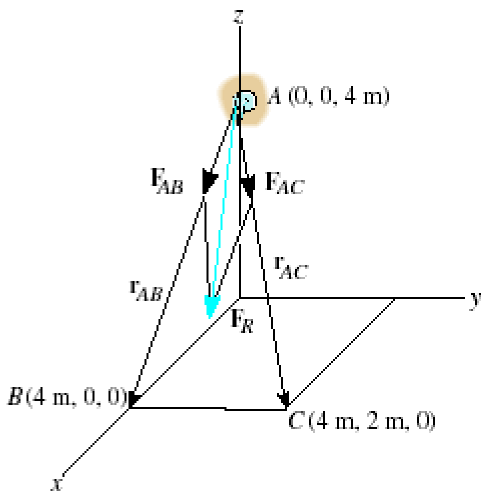
$$\overrightarrow{AB} = \underline{r_B} - \underline{r_A}$$

$$\underline{F} = F\underline{u_F} = F\left(\frac{\overrightarrow{AB}}{\overline{\overline{AB}}}\right)$$

p.62, Ex.2-15,

Determine the resultant force acting at A.





$A(0,0,4)$

$B(4,0,0)$

$C(4,2,0)$

$\overrightarrow{AB} = (4,0,-4) \cdots \overline{AB} = 5.657m$

$\overrightarrow{AC} = (4,2,-4) \cdots \overline{AC} = 6.0m$

$$\underline{F}_{AB} = F_{AB} \bullet \frac{\overrightarrow{AB}}{AB}$$

$$= \frac{100}{5.657}(4,0,-4)$$

$$= (70.71, 0, -70.71)\ N$$

$$\underline{F}_{AC} = F_{AC} \bullet \frac{\overrightarrow{AC}}{AC}$$

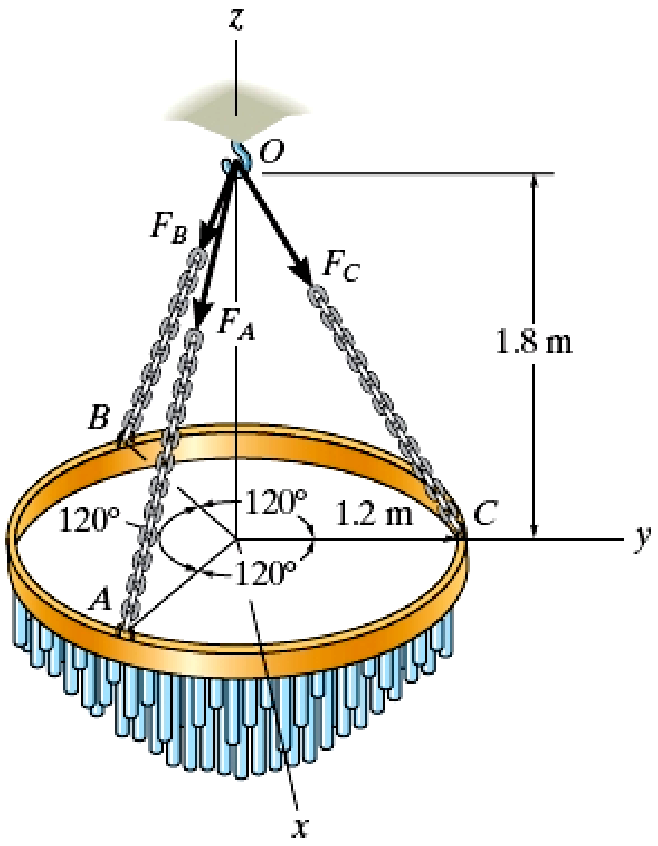
$$= \frac{120}{6}(4,2,-4)$$

$$= (80, 40, -80)\ N$$

$$\therefore \underline{R} = \underline{F}_{AB} + \underline{F}_{AC} = (150.71, 40, -150.71)\ N$$

$$R = \sqrt{150.71^2 + 40^2 + (-150.71)^2} = 217\ N$$

p.65, Problem 2-93
The chandelier is supported by three chains which are concurrent at point *O*. If the force in each chain has a magnitude of 300 N, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{F}_A = 300\frac{(1.2\cos 30°\mathbf{i} - 1.2\sin 30°\mathbf{j} - 1.8\mathbf{k})}{\sqrt{(1.2\cos 30°)^2 + (-1.2\sin 30°)^2 + (-1.8)^2}}$$

$$= \{144.1\mathbf{i} - 83.2\mathbf{j} - 249.6\mathbf{k}\}\ \text{N}$$ **Ans**

$$\mathbf{F}_B = 300\frac{(-1.2\cos 30°\mathbf{i} - 1.2\sin 30°\mathbf{j} - 1.8\mathbf{k})}{\sqrt{(-1.2\cos 30°)^2 + (-1.2\sin 30°)^2 + (-1.8)^2}}$$

$$= \{-144.1\mathbf{i} - 83.2\mathbf{j} - 249.6\mathbf{k}\}\ \text{N}$$ **Ans**

$$\mathbf{F}_C = 300\frac{(1.2\mathbf{j} - 1.8\mathbf{k})}{\sqrt{(1.2)^2 + (-1.8)^2}}$$

$$= \{166.4\mathbf{j} - 249.6\mathbf{k}\}\ \text{N}$$ **Ans**

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \{-748.8\mathbf{k}\}\ \text{N}$$

$F_R = 748.8$ N **Ans**

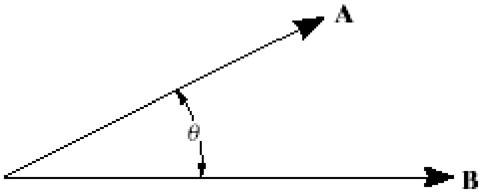
$\alpha = 90°$ **Ans**

$\beta = 90°$ **Ans**

$\gamma = 180°$ **Ans**

Dot Product

$\underline{A} \cdot \underline{B}$ **=** AB *cos* θ **,** $0^o \leq \theta \leq 180^o$



ex: Work = $\underline{\mathbf{F}} \cdot \underline{\mathbf{r}}$

- Cartesian Vector Formulation

$$\underline{i}\cdot\underline{i}=1 \qquad \underline{i}\cdot\underline{k}=0 \qquad \underline{i}\cdot\underline{j}=0$$

$$\underline{j}\cdot\underline{i}=0 \qquad \underline{j}\cdot\underline{k}=0 \qquad \underline{j}\cdot\underline{j}=1$$

$$\underline{k}\cdot\underline{i}=0 \qquad \underline{k}\cdot\underline{k}=1 \qquad \underline{k}\cdot\underline{j}=0$$

$$\underline{A}\cdot\underline{B}=(A_x\underline{i}+A_y\underline{j}+A_Z\underline{k})\cdot(B_x\underline{i}+B_y\underline{j}+B_Z\underline{k})$$

$$= A_xB_x + A_yB_y + A_zB_z$$

Dot Product

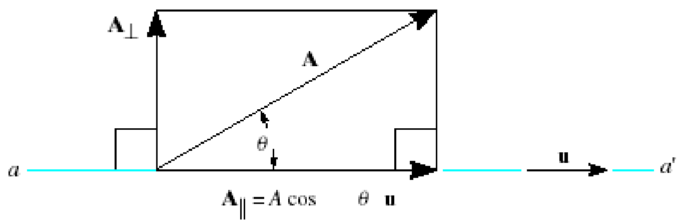
- Applications

1. angle

$\theta = \cos^{-1}(\frac{\underline{A}\cdot\underline{B}}{AB})$

when $\theta = 90^o$, $\cos\theta = 0$, $\underline{A}\cdot\underline{B} = 0 \Rightarrow A \perp B$

2. components of a vector parallel and perpendicular to a line



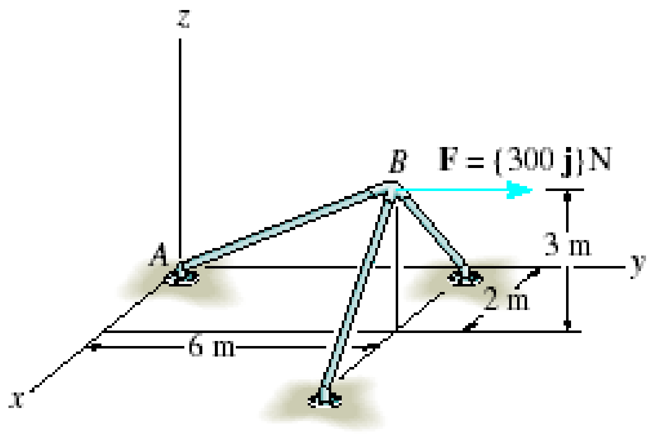
$A_{\|} = A\cos\theta = \underline{A}\cdot\underline{u}$

$\therefore \underline{A}_{\|} = A\cos\theta\cdot\underline{u} = (\underline{A}\cdot\underline{u})\cdot\underline{u}$

又 $\underline{A}_{\perp} = \underline{A} - \underline{A}_{\|}$

$A_{\perp} = A\sin\theta$

p.72, Ex. 2-17,
Determine the parallel and perpendicular components of the force to member AB.



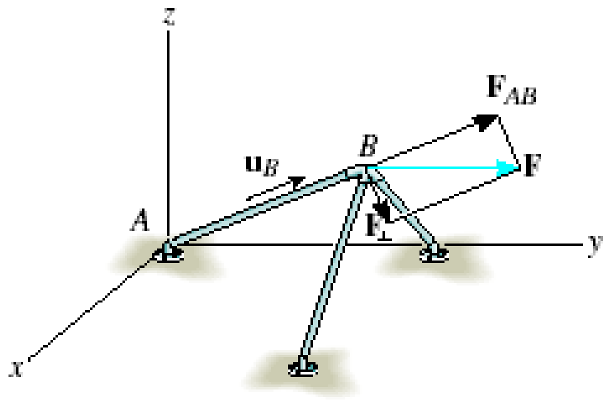


Figure 02.43(b)

$$A(0,0,0)\quad B(2,6,3)$$

$$\underline{u}_{AB} = \frac{\overrightarrow{AB}}{\overline{\overline{AB}}} = \frac{(2,6,3)}{\sqrt{2^2+6^2+3^2}} = \frac{1}{7}(2,6,3)$$

$$\underline{F}_{AB} = (\underline{F}\cdot\underline{u}_{AB})\underline{u}_{AB}$$

$$= 300\cdot\frac{6}{7}\cdot\frac{1}{7}(2,6,3)$$

$$= (73.5, 220, 110.2)\ \mathrm{N}$$

$$\underline{F}_\perp = \underline{F} - \underline{F}_{AB}$$

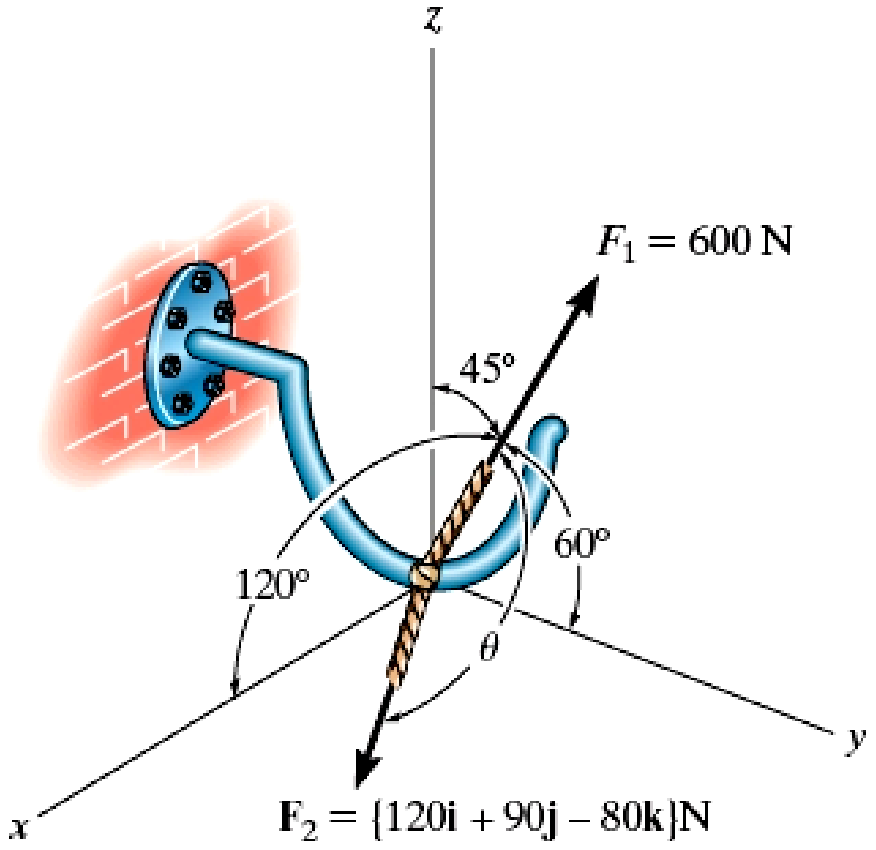
$$= (-73.5, 80, -110.2)\ \mathrm{N}$$

$$F_{AB} = \sqrt{73.5^2 + 220^2 + 110.2^2} = 257N$$

$$F_\perp = \sqrt{(-73.5)^2 + 80^2 + (-110.2)^2} = 154.7N$$

p.76, Problem 2-116

Two forces act on the hook. Determine the angle between them.Also, what are the projections of $\mathbf{F}_1$ and $\mathbf{F}_2$ along the *y* axis?



$$\mathbf{F}_1 = 600 \cos 120^\circ \mathbf{i} + 600 \cos 60^\circ \mathbf{j} + 600 \cos 45^\circ \mathbf{k}$$

$$= -300\,\mathbf{i} + 300\,\mathbf{j} + 424.3\,\mathbf{k}\,; \quad F_1 = 600\text{ N}$$

$$\mathbf{F}_2 = 120\,\mathbf{i} + 90\,\mathbf{j} - 80\,\mathbf{k}\,; \quad F_2 = 170\text{ N}$$

$$\mathbf{F}_1 \cdot \mathbf{F}_2 = (-300)(120) + (300)(90) + (424.3)(-80) = -42\,944$$

$$\theta = \cos^{-1}\left(\frac{-42\,944}{(170)(600)}\right) = 115^\circ \qquad \textbf{Ans}$$

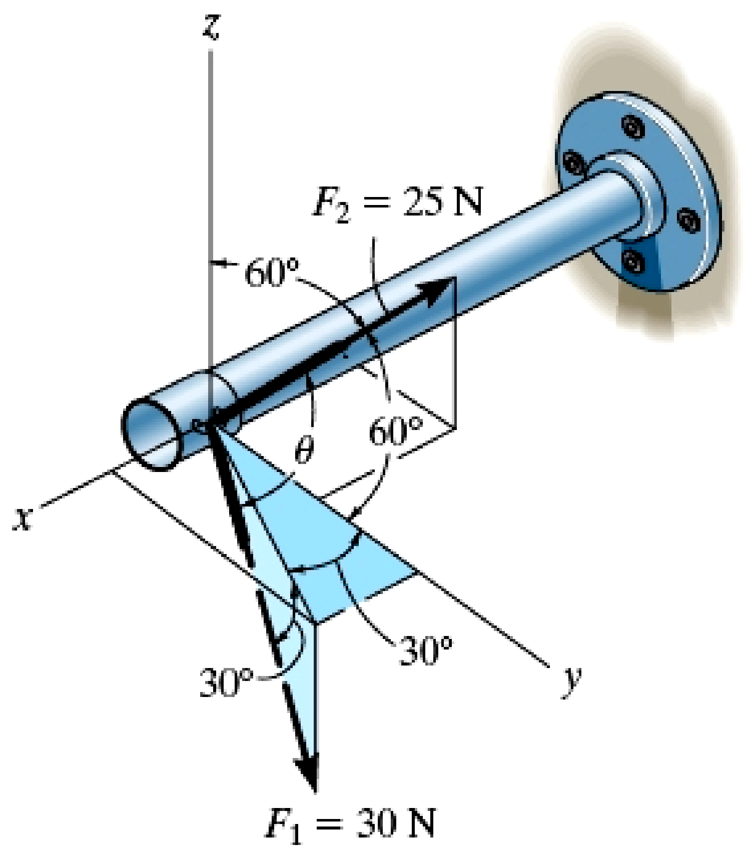
$$\mathbf{u} = \mathbf{j}$$

So,

$$F_{1y} = \mathbf{F}_1 \cdot \mathbf{j} = (300)(1) = 300\text{ N} \qquad \textbf{Ans}$$

$$F_{2y} = \mathbf{F}_2 \cdot \mathbf{j} = (90)(1) = 90\text{ N} \qquad \textbf{Ans}$$

p.78, Problem 2-133
Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



Force Vector:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30° \mathbf{i} + \cos 30° \cos 30° \mathbf{j} - \sin 30° \mathbf{k}$$
$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_1 \mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \text{ N}$$
$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\} \text{ N}$$

Unit Vector: One can obtain the angle $\alpha = 135°$ for $\mathbf{F}_2$ using Eq. 2-8. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, with $\beta = 60°$ and $\gamma = 60°$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135° \mathbf{i} + \cos 60° \mathbf{j} + \cos 60° \mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

Projected Component of $\mathbf{F}_1$ Along the Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$
$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$
$$= -5.44 \text{ N}$$

Negative sign indicates that the projected component $(\mathbf{F}_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(\mathbf{F}_1)_{F_2} = 5.44$ N. **Ans**

Thanks for your attention