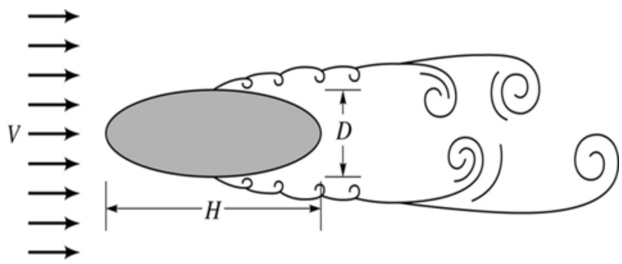
EX 7.5 A long structural component of a bridge has the cross section as shown. It is known that when a steady wind blows past this type of bluff body, vortices may develop on the downwind side that are shed in a regular fashion at some definite frequency. Since these vortices can create harmful periodic forces acting on the structure, it is important to determine the shedding frequency. For the specific structure of interest, D = 0.1 m, H = 0.3 m, and a representative wind velocity 50 km/hr. Standard air can be assumed. The shedding frequency is to be determined through the use of a small-scale model that is to be tested in a water tunnel. For the model D_m = 20 mm and the water temperature is 20℃. Determine which the test should be performed. If the shedding frequency ω for the model is found to be 49.9Hz, what is the corresponding frequency for the prototype?



【SOLUTION】

PART I

STEP 1 從題目中找出主角為 shedding frequency ω， 進而列出影響 ω 的變數包括 Length D and H、approach velocity V、 fluid density ρ、fluid viscosity μ，共六個（k=6）；即 $\omega = f(D,H,V,\rho,\mu)$ 。

STEP 2 採用 MLT 系統，以 primary dimensions 寫出六個變數的 dimensions：

$\omega \doteq T^{-1}$ $D \doteq L$ $H \doteq L$ $V \doteq LT^{-1}$ $\rho \doteq ML^{-3}$ $\mu \doteq ML^{-1}T^{-1}$ （$r = 3$）

STEP 3 由 $k - r$ 得知，會有 3 個 dimensionless groups。

STEP 4 選擇 D、V、μ 作為 repeating variables。

STEP 5~6 將 repeating variables 分別ω、H、ρ結合，寫出 dimensional group，並解出指數

$\Pi_1 = \omega D^{a_1} V^{b_1} \mu^{c_1}$ ；解出：$a_1 = 1$、$b_1 = -1$、$c_1 = 0$ → $\Pi_1 = \dfrac{\omega D}{V}$

$\Pi_2 = H D^{a_2} V^{b_2} \mu^{c_2}$ ；解出：$a_2 = -1$、$b_2 = 0$、$c_2 = 0$ → $\Pi_2 = \dfrac{H}{D}$

$\Pi_3 = \rho D^{a_3} V^{b_3} \mu^{c_3}$ ；解出：$a_3 = 1$、$b_3 = 1$、$c_1 = -1$ → $\Pi_3 = \dfrac{\rho V D}{\mu}$

STEP 7 核對每一個 dimensionless group 確為無因次。

STEP 8 寫出 3 個 dimensionless groups 的關聯 $\dfrac{\omega D}{V} = \phi\left(\dfrac{H}{D}, \dfrac{\rho V D}{\mu}\right)$

PART II

要設計一個 model，進行 model test，必須要符合那些要件？

（1）Model test 的環境與 prototype 運轉的環境，必須具有相同的 phenomenon；或者說，model test 中對於 shedding frequency 的探討，所考慮的變數一如 prototype 的運轉環境。意即，$\omega_m = f_m(D_m, H_m, V_m, \rho_m, \mu_m)$。因此，在 model test 中，透過 dimensional analysis 可獲得的 dimensionless groups 關聯亦為 $\frac{\omega_m D_m}{V_m} = \phi_m\left(\frac{H_m}{D_m}, \frac{\rho_m V_m D_m}{\mu_m}\right)$。

（2）滿足 design conditions $\frac{H}{D} = \frac{Hm}{Dm}$ and $\frac{\rho VD}{\mu} = \frac{\rho m Vm Dm}{\mu m}$。

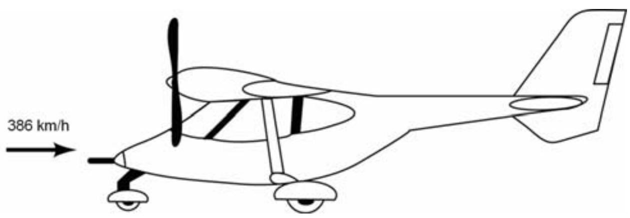
在前述兩條件符合下，才可能透過 prediction equation $\frac{\omega_m D_m}{V_m} = \frac{\omega D}{V}$ 來進行預測。

PART III

透過 design conditions 與 prediction equation 計算 model dimension H_m、model velocity V_m 及 prototype 的 shedding frequency ω

$$H_m = \frac{H}{D} Dm = ... = 60mm \qquad V_m = \frac{\mu m}{\mu}\frac{\rho}{\rho m}\frac{D}{Dm} V = ... = 4.79 m/s \qquad \omega = \frac{V}{Vm}\frac{Dm}{D}\omega m = ... = 29.0Hz$$

EX 7.7 The drag, developed on an immersed bodies, would usually be expressed with the formulation $Drag = \frac{1}{2}\rho V^2 \ell^2 \phi\left(\frac{\ell_i}{\ell}, \frac{\varepsilon}{\ell}, \frac{\rho V \ell}{\mu}\right)$. The drag on an airplane cruising at 386 km/h in standard air is to be determined from tests on a 1:10 scale model placed in a pressurized wind tunnel. To minimize compressibility effects, the airspeed in the wind tunnel is also to be 386 km/h. Determine the required air pressure in the tunnel (assuming the same air temperature for model and prototype), and the drag on the prototype corresponding to a measured force of 4 N on the model.

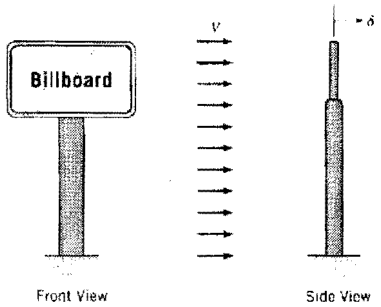


【SOLUTION】

要透過符合 geometrically similar model 預測 airplane 的阻力，兩者（model & prototype）的 Reynolds number 也必須相等；即 $\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$。

依據說明：V = V_m 且 $\frac{\ell_m}{\ell} = \frac{1}{15}$，得知 $\frac{\rho_m}{\rho} = \frac{\mu_m}{\mu}\frac{V}{V_m}\frac{\ell}{\ell_m} = 10\frac{\mu_m}{\mu}$。因此，要滿足 Reynolds numbers similarity，於 model 與 prototype 中使用相同的流體（$\rho_m = \rho$ and $\mu_m = \mu$）是辦不到的，除非在不改變 viscosity 下，提高 model test 的流體壓力至 10 倍（$\frac{p_m}{p} = \frac{\rho_m}{\rho} = 10$），即 $p_m = 10p = 10(101.3kPa) = 1{,}013kPa$。

7.53 A large, rigid, rectangular billboard is supported by an elastic column as shown. There is concern about the deflection, δ, of the top of the structure during a high wind of velocity V. A wind tunnel test is to be conducted with a 1:15 scale model. Assume the pertinent column variables are its length and diameter of cross-section, and the modulus of elasticity of the material used for the column. The only important "wind" variables are the air density and velocity. (a) Determine the model design conditions and the prediction equation for the deflection. (b) If the same structural materials are used for the model and prototype, and the wind tunnel operations under standard atmospheric conditions, what is the required wind tunnel velocity to match an 80 km/h wind?



STEP 1. 討論目標 deflection δ, 和 colum length ℓ, 直徑 d, 風密度 ρ, 風速度 V, Modulus of elasticity E 有關, 共6個 ($k=6$), 即 $\delta = f(\ell, d, \rho, V, E)$

STEP 2. 採用 FLT system, 寫出6個變數的 dimensions:

$\delta \doteq L,\ \ell \doteq L,\ d \doteq L,\ \rho \doteq FL^{-4}T^{2},\ V \doteq LT^{-1},\ E \doteq FL^{-2},\ r=3$

STEP 3. 由 $k-r=3$ 得知會有3個 dimensionless groups.

STEP 4. 選擇 ρ, V, ℓ 作為 repeating variables.

STEPS 5~6. 將 ρ, V, ℓ 分別與 δ, d, E 結合, 寫出 dimensionless groups

$\Pi_1 = \delta \rho^{a_1} V^{b_1} \ell^{c_1}$; 解出 $a_1=0, b_1=0, c_1=-1 \rightarrow \Pi_1 = \dfrac{\delta}{\ell}$

$\Pi_2 = d \rho^{a_2} V^{b_2} \ell^{c_2}$; 解出 $a_2=0, b_2=0, c_2=-1 \Rightarrow \Pi_2 = \dfrac{d}{\ell}$

$\Pi_3 = E \rho^{a_3} V^{b_3} \ell^{c_3}$; 解出 $a_3=-1, b_3=-2, c_3=0 \Rightarrow \Pi_3 = \dfrac{E}{\rho V^2}$

STEP 7. 核對 dimensionless group 確為"無因次".

STEP 8. 寫出 dimensionless groups 的關聯 $\dfrac{\delta}{\ell} = \phi\left(\dfrac{d}{\ell}, \dfrac{E}{\rho V^2}\right)$

(a) 設計 model, 進行 model test 必須滿足

① 見 Ex. 7.5

② design condition: $\dfrac{d}{\ell} = \dfrac{d_m}{\ell_m}$, $\dfrac{E}{\rho V^2} = \dfrac{E_m}{\rho_m V_m^2}$

➡ 利用 prediction equation $\dfrac{\delta}{\ell} = \dfrac{\delta_m}{\ell_m}$ 預測 δ

$\delta = \dfrac{\ell}{\ell_m}\delta_m = \dfrac{15}{1}\delta_m$

$\ell : \ell_m = 15:1$

(b) $V = 80$ Km/hr; model 與 prototype 結構材料相同

$E_m = E$; 風洞試驗在大氣壓下操作 $\rho_m = \rho$

$V_m = ?$

利用 Design condition $V_m^2 = \frac{E_m}{E} \frac{\rho}{\rho_m} V^2$

$\Rightarrow V_m^2 = V$, $V_m = 80$ Km/hr.