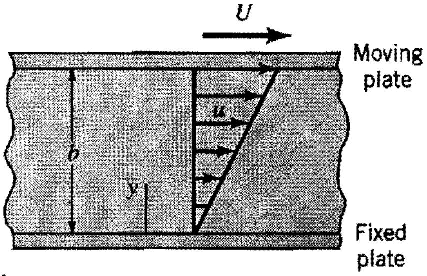
6.9 An incompressible viscous fluid is placed between two large parallel plates as shown. The bottom plate is fixed and the upper plate moves with a constant velocity, U. For these conditions the velocity distribution between the plates is linear and can be expressed as $u = U\frac{y}{b}$. Determine: (a) the volumetric dilatation rate. (b) the rotation vectors. (c) the vorticity, and (d) the rate of angular deformation.



(a) Volumetric dilatation rate $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

(b) Rotation vectors $\vec{\omega} = \omega_z \vec{k}$

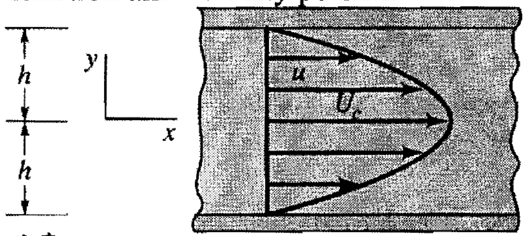
$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} = -\frac{U}{2b}\vec{k}$$

(c) Vorticity $\vec{\zeta} = 2\vec{\omega} = -\frac{U}{b}\vec{k}$ (Vorticity 為 '向量')

(d) Rate of angular deformation $\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \frac{U}{b}$ (純量)

6.37 It is known that the velocity distribution for two-dimensional flow of a viscous fluid between wide parallel plates as shown is parabolic; that is, $u = U_c\left[1 - \left(\frac{y}{h}\right)^2\right]$

With v = 0. Determine, if possible, the corresponding stream function and velocity potential.



(a) Stream function ?

由 stream function ψ 与 速度 u,v 的關係

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

可知 $u = \frac{\partial \psi}{\partial y} = U_c\left[1 - \left(\frac{y}{h}\right)^2\right] \rightarrow$ 積分 (對 y 積分)

$$\int d\psi = \int U_c\left[1 - \left(\frac{y}{h}\right)^2\right] dy \rightarrow \psi = U_c\left[y - \frac{y^3}{3h^2}\right] + f_1(x), \quad f_1(x) = ?$$

利用 $v = -\frac{\partial \psi}{\partial x} = f_1'(x) = 0$，可知 $f_1(x) =$ constant C.

因此, stream function $\psi = U_c\left(y - \frac{y^3}{3h^2}\right) + C$.

(b) velocity potential ?

由 velocity potential Φ 与 速度 u,v 的關係

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}$$

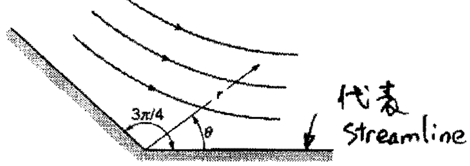
可知 $u = \frac{\partial \phi}{\partial x} = U_c\left[1 - \left(\frac{y}{h}\right)^2\right] \rightarrow$ 積分 (對x積分)

$$\int d\phi = \int U_c\left[1 - \left(\frac{y}{h}\right)^2\right] dx \rightarrow \phi = U_c\left[1 - \left(\frac{y}{h}\right)^2\right]x + f_2(y), \quad f_2(y) = ?$$

利用 $v = \frac{\partial \phi}{\partial y} = -\frac{2U_c xy}{h^2} + f_2'(y) = 0$ 可知沒有 x,y 可令 v=0

因此, 沒有 Φ 可用來描述此流場, 因此流場並非 irrotational flow.

6.47 It is suggested that the velocity potential for the incompressible, nonviscous, two-dimensional flow along the wall as shown is $\phi = r^{4/3}\cos\frac{4}{3}\theta$. Is this a suitable velocity potential for flow along the wall? Explain. 如果 ϕ is suitable，则沿著 wall 的 Stream function ψ 應為一常數



$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\partial\phi}{\partial r} = \frac{4}{3}r^{1/3}\cos\frac{4}{3}\theta$ — (1) → 積分求 ψ

$\int d\psi = \int \frac{4}{3}r^{4/3}\cos\frac{4}{3}\theta\, d\theta$ → $\psi = r^{4/3}\sin\frac{4}{3}\theta + f_1(r)$ — (1)

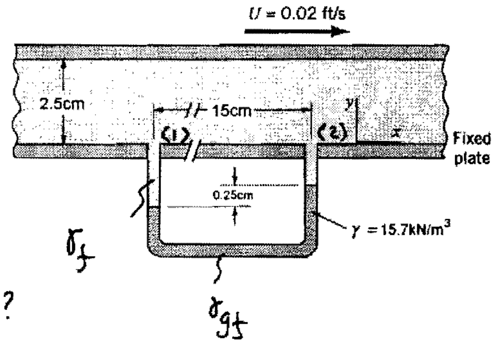
$V_\theta = -\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{4}{3}r^{1/3}\sin\frac{4}{3}\theta$ → 積分求 ψ

$\int d\psi = \int \frac{4}{3}r^{1/3}\sin\frac{4}{3}\theta\, dr$ → $\psi = r^{4/3}\sin\frac{4}{3}\theta + f_2(\theta)$ — (2)

滿足 (1) (2) $\psi = r^{4/3}\sin\frac{4}{3}\theta + C$；沿著 wall, $\theta = 0$, $\theta = \frac{3}{4}\pi$, ψ = Constant

因此，給予的 velocity potential 是合適的！

6.95 A viscous fluid (specific weight = 1.26 kN/m^3; viscosity = 1.4 N•s/m^2 is contained between two infinite, horizontal parallel plates as shown. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity U while the bottom plate is fixed. A U-tube manometer connected between two points along the bottom indicates a differential reading of 0.25 cm. If the upper plate moves with a velocity of 6×10^{-3} m/s, at what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? Assume laminar flow.



從 Navier-Stokes equation → 不可壓縮 ⊕ μ = const. →

equation 6.127 → $v = w = 0$ ⊕ $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (continuity eq.)

→ $u = u(y)$ → $0 = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right)$

$0 = -\frac{\partial p}{\partial y} - \rho g$, $0 = -\frac{\partial p}{\partial z}$

積分二次 $u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)y^2 + C_1 y + C_2$, $C_1 = ?$, $C_2 = ?$

利用 boundary condition $y = 0$, $u = 0$; $y = b$, $u = U$

得 velocity distribution $u = \frac{Uy}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - by)$

max. velocity 發生在 $\frac{\partial u}{\partial y} = 0$ 處，$\frac{\partial u}{\partial y} = \frac{U}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(2y - b)$

→ $y_m = -\frac{\mu U}{b\left(\frac{\partial p}{\partial x}\right)} + \frac{b}{2}$; $\frac{\partial p}{\partial x}$?

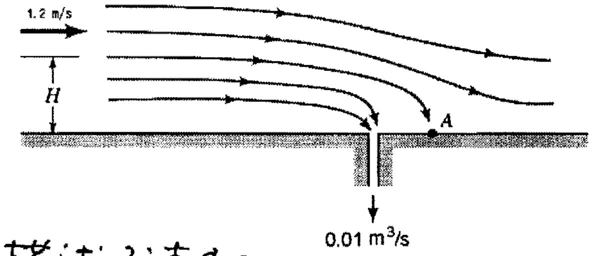
利用 manometer $p_1 + \gamma_f \Delta h - \gamma_{gf}\Delta h = p_2$, $p_1 - p_2 = (\gamma_{gf} - \gamma_f)\Delta h = 7.75\ N/m^2$

$= (15.7\ kN/m^3 - 12.6\ kN/m^3)\cdot(0.0025\ m)$

$\frac{\partial p}{\partial x} = \frac{p_1 - p_2}{\Delta x} = \frac{-7.75\ N/m^2}{0.15\ m} = -51.667\ N/m^3$

$y_m = \frac{(1.4\ N\cdot s/m^2)(6\times10^{-3}\ m/s)}{(0.0025\ m)(51.667\ N/m^3)} + \frac{0.025\ m}{2} = 1.9\ cm$

6.54 Water flows over a flat surface at 1.2 m/s as shown. A pump draws off water through a narrow slit at volume rate of 0.01 m^3/s per meter length of the slit. Assume that the fluid is incompressible and inviscid and can be expressed by the combination of a uniform flow and a sink. Locate the stagnation point on the wall (point A) and determine the equation for the stagnation streamline. How far above the surface, H, must the fluid be so that it does not get sucked into the slit?



已知 $\psi_{uniform} = Ur\sin\theta$

$\psi_{sink} = -\frac{m}{2\pi}\theta$

$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$, $V_\theta = -\frac{\partial \psi}{\partial r}$

由 uniform flow與 sink 結合的流場如何描述?速度?

$\psi = \psi_{uniform} + \psi_{sink} = Ur\sin\theta - \frac{m}{2\pi}\theta$

$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = U\cos\theta - \frac{m}{2\pi r}$, $V_\theta = -\frac{\partial \psi}{\partial r} = -U\sin\theta$

沿著 wall, $\theta = 0$、π, $V_\theta = 0$; $r_{stagnation}$處, 出現 stagnation, $V_r = 0$

由 $V_r = U\cos\theta - \frac{m}{2\pi r}$ 得知 $r_{stagnation}$ 在 $\theta = 0$, $\frac{m}{2\pi U}$ 處。

即 $r_{stagnation} = \frac{m}{2\pi U} = \frac{0.02\ m^3/s}{2\pi\ (1.2\ m/s)} = 2.653\ m$ (slit右側, wall)

其中,由slit排開的水流量為 sink 的 strength 的一半,因此 $m = 0.02\ m^3/s$

至於經過 stagnation point 的 streamline 的方程式如何取得?

先檢視在 stagnation point處的 stream function?

$\psi_{\theta=0^\circ,\ r=\frac{m}{2\pi U}} = 0$

因此,經過 stagnation point 的 streamline 的方程式為 $Ur\sin\theta - \frac{m}{2\pi}\theta = 0$

令 $r\sin\theta = y$, 則 streamline 方程式可寫成 $y = \frac{m}{2\pi U}\theta$

該方程式可視為 solid boundary, 上方的流體不會穿越, 流入slit中。

距離 wall 有多遠? max. distance H 發生在 $\theta \to \pi$處,

即 $H = \frac{m}{2\pi U}\cdot\pi = \frac{m}{2U} = \frac{0.02\ m^3/s}{2\times 1.2\ m/s} = 8.33\times 10^{-3}\ m$