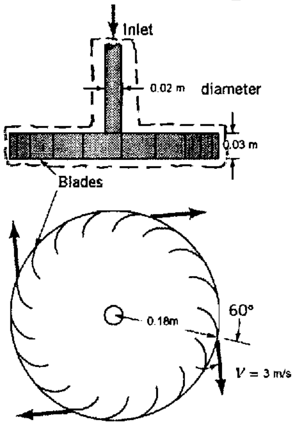
5.6 Water flows out through a set of thin, closely spaced blades as shown with a speed of V = 3 m/s around the entire circumference of the outlet. Determine the mass flowrate through the inlet pipe.



。選定 Control volume，含括 blades 及進、出口。

• 依質量守恆原理 $\dot{m}_{inlet} = \dot{m}_{outlet}$

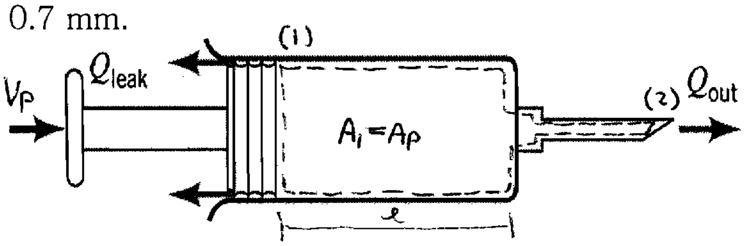
$$\dot{m}_{outlet} = \rho A_{outlet} V_{outlet} \cos 60^\circ$$

$$= \rho (2\pi) r_{outlet} h V_{outlet} \cos 60^\circ$$

$$= (10^3 \, kg/m^3)(2\pi)(0.18\, m)(0.03\, m)(3\, m/s) \cos 60^\circ$$

$$= 51 \, kg/s$$

5.29 A hypodermic syringe as shown is used to apply a vaccine. If the plunger is moved forward at the steady rate of 20 mm/s and if vaccine leaks past the plunger at 0.1 of the volume flowrate out the needle open, calculate the average velocity of the needle exit flow. The inside diameters of the syringe and the needle are 20 mm and 0.7 mm.



選定 control volume，其体積隨 plunger 移動而改變，屬於 deforming C∀。

依 continuity equation

$$\frac{\partial}{\partial t}\int_{C\forall} \rho \, d\forall + \dot{m}_2 + \rho Q_{leak} = 0 \text{；其中，} \int \rho \, d\forall = \rho (A\ell_1 + \forall_{needle}) \quad (1)$$

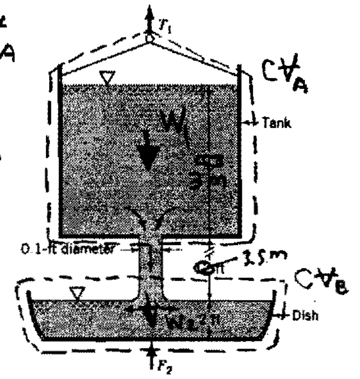
$$\frac{\partial}{\partial t}\int_{C\forall} \rho \, d\forall = \rho A_1 \frac{\partial \ell}{\partial t} = -\rho A_1 V_p \quad (2)$$

代(2)入(1)，得 $-\rho A_1 V_p + \rho Q_2 + \rho Q_{leak} = -\rho A_1 V_p + \rho A_2 V_2 + \rho (0.1) A_2 V_2 = 0 \quad (3)$

由(3)，得 $1.1 A_2 V_2 = A_1 V_p$，$V_2 = \frac{A_1}{A_2} \cdot \frac{V_p}{1.1} = \left(\frac{d_1}{d_2}\right)^2 \frac{V_p}{1.1}$

$$= \left(\frac{20\, mm}{0.7\, mm}\right)^2 \frac{20\, mm/s}{1.1} = \cdots = 14.8\, m/s$$

5.52 Water flows from a large tank into a dish as shown. (a) If at the instant shown the tank and the water in it weigh W_1 N, what is the tension, T_1, in the cable supporting the tank? (b) If at the instant shown the dish and the water in it weigh W_2 N, what is the force, F_2, needed to support the dish?



Vertical-component of linear momentum equation - CV_A

$-V_{out}\,\rho\,V_{out}\,A_{out} = T_1 - W_1 \quad (1)$

其中, $V_{out} = \sqrt{2g\,h_A} = \sqrt{2\cdot\frac{9.8\,m}{s^2}\cdot 3m} = 7.7\,m/s \quad (2)$

代(2)入(1),得 $T_1 = W_1 - 41.91\,N$

Vertical-component of linear moment equation - CV_B

$V_{inlent\ of\ CV_B}\ \dot{m}_{inlet\ of\ CV_B} = F_2 - W_2 \quad (3)$

其中, $V_{inlet\ of\ CV_B} = \sqrt{2g(h_A + h_B)} = \sqrt{2\ \frac{9.81\,m}{s^2}\ (3m + 3.5m)} = 11.3\,m/s \quad (4)$

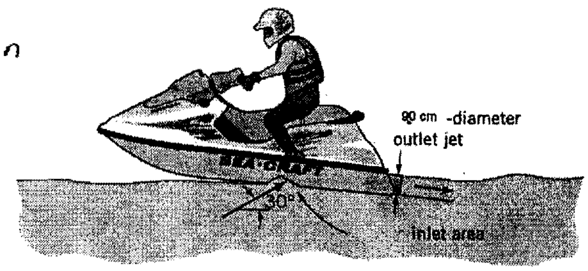
$\dot{m}_{inlet\ of\ CV_B} = \dot{m}_{outlet\ of\ CV_A} = \rho\,V_{out}\,A_{out} = (10^3\,\frac{Kg}{m^3})(7.7\,\frac{m}{s})(\frac{\pi}{4}(0.03\,m)^2) \quad (5)$

代(4)(5)入(3) $= 5.443\,Kg/s$

$F - W_2 = 61.504\,N$

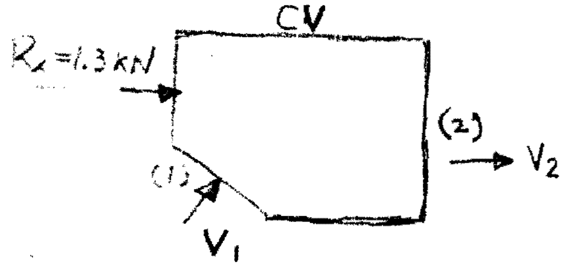
$F = W_2 + 61.504\,N$

5.66 The thrust developed to propel the jet ski don as shown is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 1.3 kN thrust? Assume the inlet and outlet jets of water are free jets.



X-component of linear momentum equation

$$\underbrace{\cancel{\frac{\partial}{\partial t}\int u\rho\, d\forall}}_{steady}^{0} + \int_{cs} u\,\rho\,\vec{V}\cdot\hat{n}\,dA = \Sigma F_x$$



$(V_1\cos 30)\,\rho(-V_1)A_1 + V_2\,\rho\,V_2 A_2 = R_x \quad (1)$

Continuity equation $\quad \dot{m} = \rho V_1 A_1 = \rho V_2 A_2 \quad (2)$

代(2)入(1),得 $R_x = \dot{m}\,(V_2 - V_1\cos 30)$

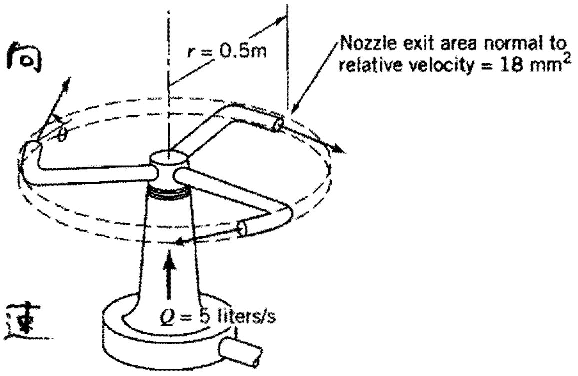
$= \rho V_1 A_1\,(V_2 - V_1\cos 30^\circ)$

$= \rho V_1^2 A_1\,(\frac{V_2}{V_1} - \cos 30^\circ)$ 其中, $\frac{V_2}{V_1} = \frac{A_1}{A_2} = \frac{160\,cm^2}{\frac{\pi}{4}(9\,cm)^2} = 2.52$

$= \rho V_1^2 A_1\,(2.52 - \cos 30^\circ)$

$V_1 = \left[\frac{R_x}{\rho A_1 (2.52 - \cos 30)}\right]^{1/2} = 7\,m/s \quad ; Q_1 = A_1 V_1 = 0.112\,m^3/s$

5.76 Five liters/s of water enter the rotor along the axis of rotation as shown. The cross-sectional area of each the three nozzle exits normal to the relative velocity os 18 mm^2. How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting torque is reduced to zero and (a) $\theta = 0°$, (b) $\theta = 30°$, (c) $\theta = 60°$?



由 moment-of-momentum equation之軸向分量得知

$$T_{shaft} = -r_2 \dot{m} V_{\theta 2}$$

其中，$V_{\theta 2}$=流体離開噴嘴之絕對速度在切線θ之分量。

$$= W_{\theta 2} - U (= r_2 \omega)$$

$W_{\theta 2}$= 流体離開噴嘴之相對於轉動之噴嘴之相對速度在切線θ之分量。

U = 轉動之噴嘴之轉動速度 $r_2\omega$。

$$W_{\theta 2} = \frac{Q}{3A_{nozzle}} \cos\theta$$

因此 $$T_{shaft} = -\rho Q r_2 \left(\frac{Q\cos\theta}{3A_{nozzle}} - r_2\omega \right)$$

(a) 當 $\theta = 0°$, $1\ liter = 10^{-3}\ m^3$

① The resisting torque required to hold the rotor stationary?

即 $\omega = 0$, $T_{shaft}\big|_{required} = +\rho Q r_2 \frac{Q\cos\theta}{3A_{nozzle}}$

$$= (10^3\ kg/m^3)(5\times10^{-3}\ m^3/s)^2(0.5m)/(3)(18\times10^{-6}\ m^2)$$

$$= 231.481\ N\cdot m$$

② The resisting torque is zero

$$\omega = \frac{Q\cos\theta}{3 r_2 A_{nozzle}} = \frac{5\times10^{-3}\ m^3/s}{3\times(0.5m)(18\times10^{-6}\ m^2)}$$

$$= 185.185\ rad/s$$

(b) 當 $\theta = 30^\circ$

① The resisting torque required to hold the rotor stationary?

即 $\omega = 0$ $\quad T_{\substack{shaft \\ required}} = \rho Q r_2 \dfrac{Q\cos\theta}{3A_{nozzle}}$

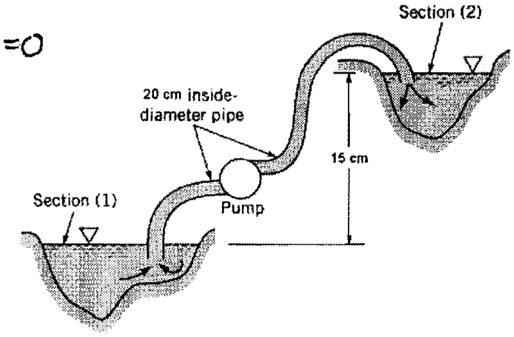
$$= \frac{(10^3\,kg/m^3)(5\times10^{-6}\,m^3/s)(0.5\,m)\cos 30^\circ}{3(18\times10^{-6}\,m^2)}$$

$= 200.468\ N\cdot m$

② The resisting torque is zero

$$\omega = \frac{Q\cos\theta}{3 r_2 A_{nozzle}} = 160.375\ rad/s$$

5.121 Water is to be moved from one large reservoir to another at a higher elevation as shown. The loss of available energy associated with 0.07 m^3/s being pumped from section (1) to (2) is loss = 61 $V^2/2\ m^2/s^2$, where V is the average velocity of water in the 20 cm inside diameter piping involved. Determine the amount of shaft power required.



$P_1 = P_2$, $V_1 = V_2$, No heat transfer $\dot{Q}_{net\,in} = 0$

Energy equation 可簡化

$$\dot{W}_{\substack{shaft \\ net\,in}} = \dot{m}\, w_{\substack{shaft \\ net\,in}}$$

$$= \dot{m}\left[g(z_2 - z_1) + loss\right] = \rho Q\left[g(z_2 - z_1) + 61V^2/2\right]$$

其中, $V = \dfrac{Q}{A} = \dfrac{0.07\,m^3/s}{\frac{\pi}{4}(0.2\,m)^2} = 2.228\ m/s$

因此 $\dot{W}_{\substack{shaft \\ net\,in}} = (10^3\,kg/m^3)(0.07\,m^3/s)\left[(9.81\,m/s^2)(0.15\,m) + 61(2.228\,m/s)^2/2\right]$

$= 10.804\ KW$